


**EU Twinning Project JO/13/ENP/ST/23**  
**Component 2: Sampling Techniques**

**Training in Seasonal Adjustment**  
**Amman, 18-22 May 2014**

R. Iannaccone (Istat)  
 N.Stuttard (NI-CO)

**GENERAL INTRODUCTION**



**Scope of the course**

**Conduct a training course in seasonal adjustment**

With the Purpose

1. To train the DoS staff in seasonal adjustment methodologies
2. To discuss software solutions for the seasonal adjustment

Reaching the output

1. Knowledge gained on the state of the art of seasonal adjustment methods: methodology (X-12-Arima and Tramo-Seats) and software (JDemetra+)
2. Transfer the European Union, experience in seasonal adjustment
3. Recommendations prepared on how to implement seasonal adjustment calculations in DoS.



**Eurostat Press Release - 1**



72/2014 - 6 May 2014


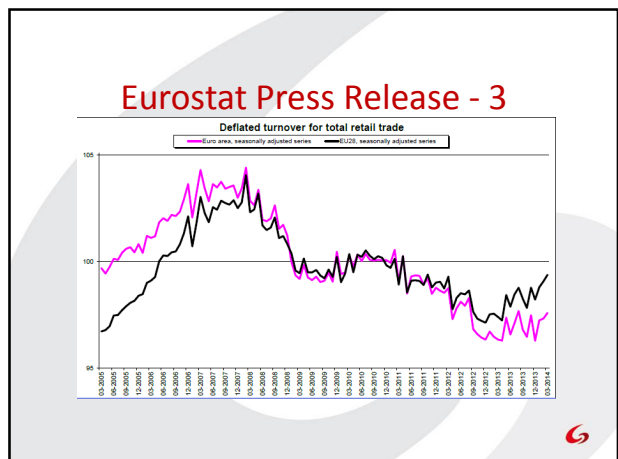
March 2014 compared with February 2014  
**Volume of retail trade up by 0.3% in both euro area and EU28**



**Eurostat Press Release - 2**

In March 2014 compared with February 2014, the seasonally adjusted volume of retail trade<sup>1</sup> rose by 0.3% in both the euro area<sup>2</sup> (EA18) and the EU28<sup>3</sup>, according to estimates from Eurostat, the statistical office of the European Union. In February<sup>3</sup> retail trade increased by 0.1% and 0.3% respectively.

In March 2014 compared with March 2013<sup>4</sup> the retail sales index increased by 0.9% in the euro area and by 1.6% in the EU28.

## Roberto Iannaccone

1. Currently in charge of the Survey on Indices of Turnover in Service Sector (from sampling to estimation methods) and the quarterly press release. Coordinator of Istat Center of Competence for seasonal adjustment
2. Short-Term Statistics Coordinator for Italy and Member of the Eurostat Task Force on Index of Services production
3. Participation to several twinning projects (e.g. Course on Seasonal Adjustment in Sarajevo)
4. Time series Analysis (business cycle analysis) and estimation methods for short term statistics (calibration techniques)



## Nigel Stuttard

1. Retired
2. Previously with ONS for 25 years working on retail prices, labour market and national accounts.
3. Head of Time Series Analysis branch from 2004-2012
4. Member of Eurostat Steering Group on Seasonal Adjustment and co-author of EU Guidelines on Seasonal Adjustment.
5. Participation in twinning project in Lebanon



## Organisation of the course

1. Overview of time series analysis
2. Why to seasonally adjust time series
3. How to carry out seasonally adjust time series:
  - Methodology: X12-Arima and Tramo-Seats
  - Software: JDemetra+
4. Using JDemetra+:
  - How to create input
  - How to read output
5. Recommendations



## Part 1 Time series Analysis

1. Basic Knowledge (Mean, Variance and Correlation)
2. Typical Plots for time series analysis
3. How to read a time series analysis
4. Operators for time series analysis

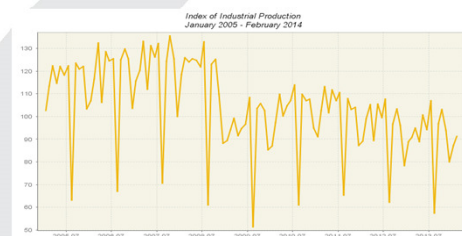


## Data types

1. Cross-section:
  - N** Observations for **N** individuals  
(Height of students in the classroom)
2. Time series
  - T** Observations collected at different fixed points in a time span  
(Monthly arrival of tourists)
3. Panel data
  - Dataset of dimension **N x T**



## Time series



## General Definition

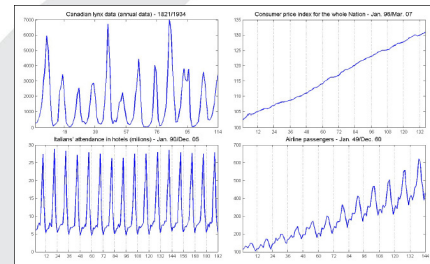
### Important features of time series

- Memory or persistence
- Sequential observations non-interchangeable
- Observed times series: a sample realisation

A sample of T successive observations in time is not a realisation of T different random variables but the realisation of a single stochastic process, the **memory** of which is given by the **degree of dependency** between the composing random variables



## Different type of time series



## Time series: graphical representation

1. Seasonal Plot
2. Sub-series Plot
3. Correlogram

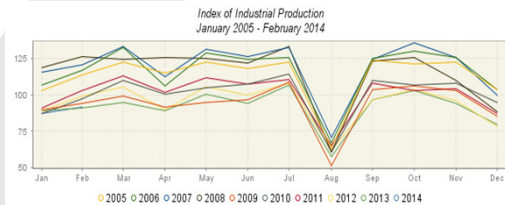


## IPI

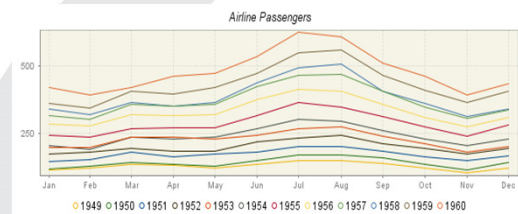
	jan	feb	mar	apr	may	jun	jul	aug	sep	oct	nov	dec
2005	102.6	113.4	122.4	114.5	122.1	118.1	122.4	63.1	123.6	120.8	122.1	103.3
2006	106.9	116.5	132.5	106.1	138.5	124.5	125.3	66.9	124.9	129.7	125.5	103.5
2007	115.5	120.3	133.1	112	131.2	126.1	132.2	70.7	124.2	135.4	125.3	100
2008	118.4	126	124	125.4	124.6	121.8	132.9	61.1	123.1	125.2	109.7	88.2
2009	89.4	94.2	99.3	91.5	94.6	96.5	106.3	51.5	103.5	105.8	102.6	85.2
2010	86.9	97.3	109.8	100.1	104.8	107	113.9	61	109.7	106.9	107.6	95
2011	91.1	102.6	113.2	101.7	111.8	107	110.5	65.3	107.8	103	104	87.1
2012	89.2	99	105.3	89.4	105.4	99.5	107.6	62.2	96.5	103.4	96	78.4
2013	89	90.9	94.8	89	100.6	94.2	106.8	57.5	96.7	103	93.9	79.9
2014	87.3	91.3										



## Seasonal Plot: IPI



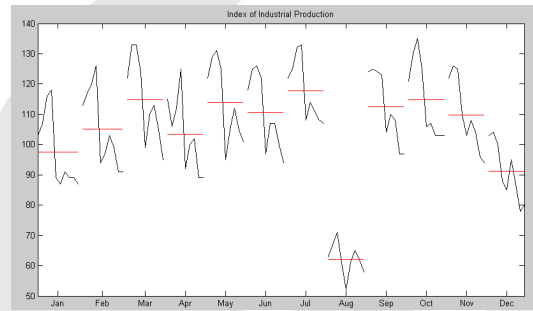
## Seasonal Plot: Airline Passengers



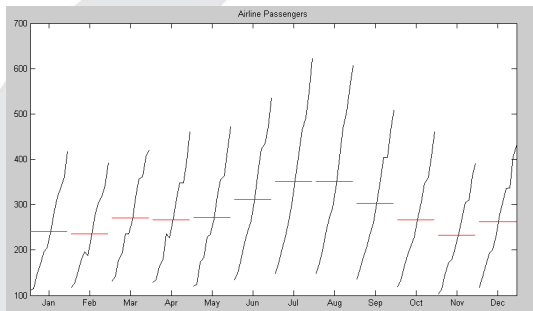
### IPI

	jan	feb	mar	apr	may	jun	jul	aug	sep	oct	nov	dec
2005	102.6	113.4	122.4	114.5	122.1	118.1	122.4	63.1	123.6	120.8	122.1	103.3
2006	106.9	116.5	132.5	106.1	128.5	124.5	125.3	66.9	124.9	129.7	125.5	103.5
2007	115.5	120.3	133.1	112	131.2	126.1	132.2	70.7	124.2	135.4	125.3	100
2008	118.4	126	124	125.4	124.6	121.8	132.9	61.1	123.1	125.2	109.7	88.2
2009	89.4	94.2	99.3	91.5	94.6	96.5	108.3	51.5	103.5	105.8	102.6	85.2
2010	86.9	97.3	109.8	100.1	104.8	107	113.9	61	109.7	106.9	107.6	95
2011	91.1	102.6	113.2	101.7	111.8	107	110.5	65.3	107.8	103	104	87.1
2012	89.2	99	105.3	89.4	105.4	99.5	107.6	62.2	96.5	103.4	96	78.4
2013	89	90.9	94.8	89	100.6	94.2	106.8	57.5	96.7	103	93.9	79.9
2014	87.3	91.3										

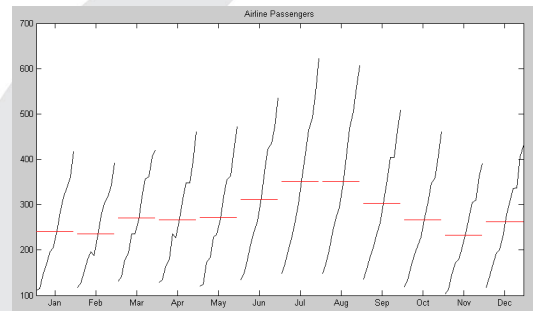
### Subseries Plot: IPI



### Subseries Plot: Airline Passengers



### Subseries Plot: Airline Passengers



### Time series: some formulas

1. Mean

$$\mu = \frac{\sum_{t=1}^T x_t}{T}$$

2. Variance

$$\sigma^2 = \frac{\sum_{t=1}^T (x_t - \mu)^2}{T}$$

3. Autocorrelation

$$\gamma_k = \frac{\sum_{t=1}^T (x_t - \mu)(x_{t-k} - \mu)}{\sigma^2}$$

### Autocorrelation

Autocorrelation: refers to the correlation of a time series with its own past and future values. Positive autocorrelation might be considered a specific form of "persistence", a tendency for a system to remain in the same state from one observation to the next.

$$k=1 \quad \gamma_1 = \frac{\sum_{t=2}^T (x_t - \mu)(x_{t-1} - \mu)}{\sigma^2}$$

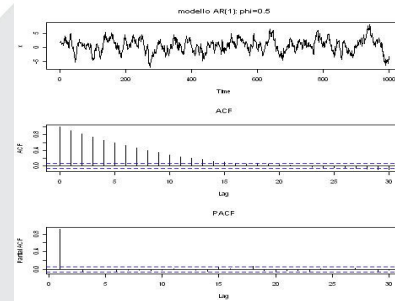
## Correlogram

An important guide to the persistence in a time series is given by the series of quantities called the sample autocorrelation coefficients, which measure the correlation between observations at different times.

The set of autocorrelation coefficients arranged as a function of separation in time is the sample autocorrelation function or the acf.



## Correlogram: an example



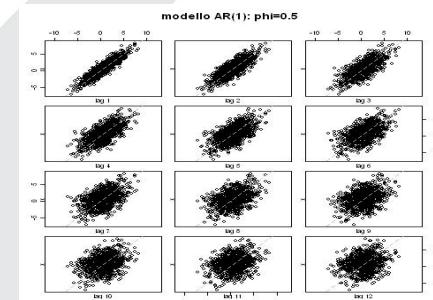
## Lagged Scatter Plot

The simplest graphical summary of autocorrelation in a time series is the lagged scatterplot, which is a scatterplot of the time series against itself offset in time by one ( $k=1$ ) to several time steps ( $k=12$ )

A random scattering of points in the lagged scatterplot indicates a lack of autocorrelation.



## Lagged Scatter Plot: an example



## Stationarity

Chapman (2004) introduces the idea of stationarity from an intuitive point of view:

“Broadly speaking, a time series is said to be stationary if there is no systematic change in mean (no trend), if there is no systematic change in variance and if strictly periodic variations have been removed. In other words, the properties of one section of the data are much like those of any other section.”



## Stationarity: in other terms

Weakly stationarity:

- Mean independent of time
- Variance independent of time
- Covariance dependent only on the difference between lags and not from time  $t$



### Transformation inducing stationarity

Mean independent of time:

- Regular difference:  $Z_t = X_t - X_{t-1}$
- Seasonal difference:  $Z_t = X_t - X_{t-s}$   
 s=4 (quarterly series) or s=12 (monthly series) to deal with non stationarity in seasonal time series

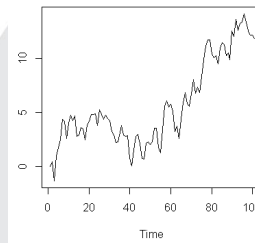
➤ Variance independent of time

Logarithm:  $Z_t = \ln(X_t)$

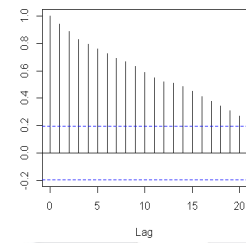


### Non Stationarity in mean: example

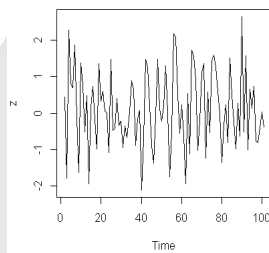
Mean Stationarity



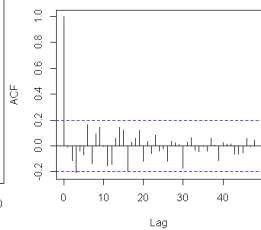
ACF



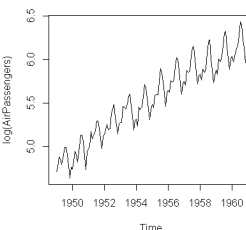
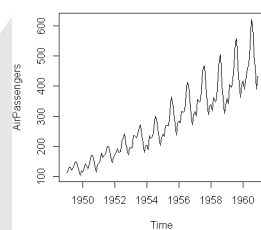
### Non Stationarity in mean: transformation



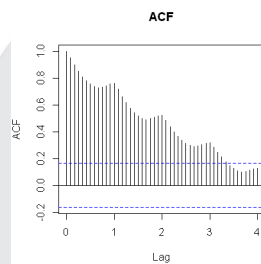
ACF



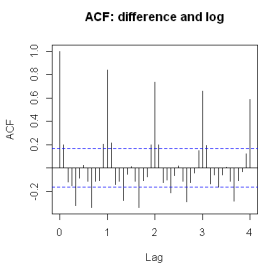
### Non Stationarity in variance: example



### Non Stationarity in variance and regular difference.....



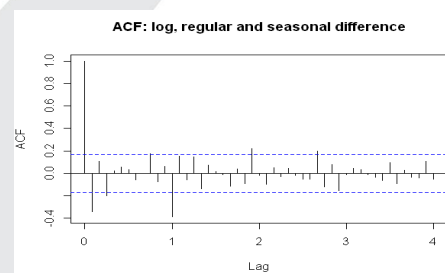
ACF



ACF: difference and log



### ..... and seasonal difference



ACF: log, regular and seasonal difference



## Backshift notation

$$BX_t = X_{t-1}$$

- Backshift operator  $B^2 X_t = B(BX_t) = BX_{t-1} = X_{t-2}$

$$B^s X_t = X_{t-s}$$

- Regular Difference  $X_t - X_{t-1} = X_t - BX_t = (1-B)X_t = \nabla X_t$

- Second Order Difference

$$\nabla^2 X_t = (1-B)^2 X_t$$

- Second Difference

$$(1-B^2)X_t$$

- Seasonal difference:  $(1-B^s)X_t = \nabla_s X_t$



## Part 2

Transfer the European Union experience in seasonal adjustment:

- STS Regulation
- Eurostat Guidelines on Seasonal Adjustment Method



## Eurostat Database

<ul style="list-style-type: none"> <li>• General and regional statistics               <ul style="list-style-type: none"> <li>• Regions and cities (including metropolitan regions)</li> <li>• Maritime regions (coastal regions)</li> <li>• Degree of urbanisation</li> <li>• Land cover/use statistics (LUCA2)</li> <li>• Rural development</li> <li>• Cohesion policy indicators</li> </ul> </li> <li>• International cooperation:               <ul style="list-style-type: none"> <li>• Enlargement countries</li> <li>• European Neighbourhood Policy countries</li> <li>• International statistical cooperation</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>• Economy and finance               <ul style="list-style-type: none"> <li>• National accounts (including GDP)</li> <li>• ESA 95 Input-Output tables</li> <li>• European sector accounts</li> <li>• Government finance statistics</li> <li>• Exchange rates</li> <li>• Interest rates</li> <li>• Harmonized Indices of Consumer Prices (HICP)</li> <li>• Purchasing power parities (PPPs)</li> <li>• Balance of payments</li> </ul> </li> <li>• Agriculture and fisheries               <ul style="list-style-type: none"> <li>• Agriculture</li> <li>• Forestry</li> <li>• Fisheries</li> <li>• Organic farming</li> <li>• Agri-Environmental Indicators</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>• Population and social conditions               <ul style="list-style-type: none"> <li>• Population</li> <li>• Health (including health and safety at work)</li> <li>• Education and training</li> <li>• Labour market (including Labour Force Surveys (LFS))</li> <li>• Income, Social Inclusion and Living conditions</li> <li>• Social protection</li> <li>• Household Budget Surveys</li> <li>• Youth</li> <li>• Crime and criminal justice</li> <li>• Culture</li> <li>• Quality of life indicators</li> </ul> </li> <li>• International trade               <ul style="list-style-type: none"> <li>• International trade</li> </ul> </li> </ul>
<ul style="list-style-type: none"> <li>• Industry, trade and services               <ul style="list-style-type: none"> <li>• Structural business statistics</li> <li>• Short-term business statistics</li> <li>• Tourism</li> <li>• Manufactured goods (Prodcom)</li> <li>• Information society</li> <li>• Postal services</li> </ul> </li> </ul>		



## STS Domain

### Introduction

#### What are short-term business statistics?

Short-term statistics (STS) describe the most recent developments of European economies. STS cover four major economic domains: industry, construction, retail trade and other services (for example transport, information and communication, business services but not financial services).

In the field of STS, the development in the different economic domains is described with a series of indicators (STS indicators) such as production, turnover, new orders received, prices, number of persons employed, gross wages and several more.

STS indicators are published as indices which show the changes of the indicator in comparison with a fixed reference year (currently 2005). The indicators do not represent absolute amounts or monetary values.



## Regulation

Regulation No 1158/2005 of the European Parliament and of the council amending Council Regulation No 1165/98

- Collection of data
- Periodicity
- Level of detail
- Processing
- Transmission and Form
- Treatment of confidential data



## Form

### Form

The text under heading (d) (Form) is replaced by the following:

1. All of the variables are to be transmitted in unadjusted form, if available.
2. In addition, the production variable (No 110) and the hours-worked variable (No 220) are to be transmitted in working-day adjusted form. Wherever other variables show working-day effects, Member States may also transmit those variables in working-day adjusted form. The list of variables to be transmitted in working-day adjusted form may be amended in accordance with the procedure laid down in Article 18.
3. In addition, Member States may transmit the variables seasonally adjusted and may also transmit the variables in the form of trend cycles. Only if data are not transmitted in these forms, may the Commission (Eurostat) produce and publish seasonally adjusted and trend-cycle series for these variables.



## Eurostat activities

### Seasonal adjustment

- Revised ESS guidelines on seasonal adjustment
- Handbook on seasonal adjustment
- JDemetra +



## Guidelines revision

2009 guidelines on seasonal adjustment widely accepted and implemented at ESS level

A recognised international reference for

- a) Statistical officer apprentices in the field
- b) Experts of seasonal adjustment
- c) Researchers



## New Needs

Need of going into more details concerning both the pre-treatment and the seasonal adjustment for the sake of clarity and completeness

Need for a clearer distinction between methods and software incorporating the recent changes related to methods and software for seasonal adjustment



## Structure

Guidelines structured in 7 sections

- **Covering all phases of seasonal adjustment**

Each section of the guidelines structured in a number of items

- **Covering various steps within a given phase**

Items presented within an harmonised template



## Items Structure

**Description:** synthetic presentation of the topic addressing main relevant issues

**Options:** bulleted list of possible options to be used

**Alternatives:** ranked alternatives

- a) most recommended
- b) acceptable
- c) to be avoided



## Index

<b>1 – PRE-TREATMENT</b>
Item 1.1: Objectives of the pre-treatment of the series
Item 1.2: Graphical analysis of the series
Item 1.3: Calendar adjustment
Item 1.3.1: Methods for finding working day adjustment
Item 1.3.2: Correction for moving holidays
Item 1.3.3: National and EU-wide area calendars
Item 1.4: Outlier detection and correction
Item 1.5: Model selection
Item 1.6: Decomposition scheme
<b>2 – SEASONAL ADJUSTMENT</b>
Item 2.1: Choice of seasonal adjustment approach
Item 2.2: Consistency between raw and seasonally adjusted data
Item 2.3: Direct versus indirect approach
Item 2.3.1: Direct versus indirect approach: dealing with data from different agencies
<b>3 – REVISION POLICIES</b>
Item 3.1: General revision policy
Item 3.2: Concurrent versus current adjustment
Item 3.3: Horizons for published revisions
<b>4 – QUALITY OF SEASONAL ADJUSTMENT</b>
Item 4.1: Validation of seasonal adjustment
Item 4.2: Quality measures for seasonal adjustment
Item 4.3: Comparing alternative approaches and strategies
Item 4.4: Metadata template for seasonal adjustment
<b>5 – SPECIFIC ISSUES ON SEASONAL ADJUSTMENT</b>
Item 5.1: Seasonal adjustment of short time series
Item 5.2: Treatment of problematic series
<b>6 – DATA PRESENTATION ISSUES</b>





## Graphical analysis of the series

Detailed graphical analysis in time domain based on basic graphs and autocorrelograms previous to perform seasonal adjustment

To be used in the pre-treatment of time series to individuate outlier and calendar effects



## Choice of seasonal adjustment approach

TRAMO-SEATS and X-12-ARIMA

choice can be based on past experience subjective appreciation and characteristics of time series



## Key issues: revision

Revision policies of seasonally adjusted data due to revisions of the unadjusted data (raw) and to a better estimate of seasonal pattern after new information is available

Well defined and publically available revision policy and release calendar

Revision period of the seasonally adjusted data must at least cover the extent of the raw data revision period



## Key issues: quality

Use a detailed set of graphical, descriptive, non-parametric and parametric criteria to validate the seasonal adjustment.

Re-do the seasonal adjustment with a different set of options in case of non-acceptance of results.



## Key issues: quality and JDemetra+

Particular attention must be paid to the following suitable characteristics of seasonal adjustment series:

- absence of residual seasonality
- absence of residual calendar effects
- absence of an over-adjustment of seasonal and calendar effects
- absence of significant and positive autocorrelation for seasonal lags in the irregular component
- stability of the seasonal component



## Part 3

### Why seasonally adjustment?

1. Growth rates
2. Unobserved components
3. Deterministic effects



### Growth rates

In short-term statistics two growth rates are considered

-On the same period of the previous year:

$$G_y = 100 \frac{X_t - X_{t-12}}{X_t}$$

-On the previous period

$$G_m = 100 \frac{X_t - X_{t-1}}{X_t}$$



### Growth rates: IPI

Month	Raw		G_y	G_m
	2012	2013		
jan	89,2	89	-0,2%	13,5%
feb	99	90,9	-8,2%	2,1%
mar	105,3	94,8	-10,0%	4,3%
apr	89,4	89	-0,4%	-6,1%
may	105,4	100,6	-4,6%	13,0%
jun	99,5	94,2	-5,3%	-6,4%
jul	107,6	106,8	-0,7%	13,4%
aug	62,2	57,5	-7,6%	-46,2%
sep	96,5	96,7	0,2%	68,2%
oct	103,4	103	-0,4%	6,5%
nov	96	93,9	-2,2%	-8,8%
dec	78,4	79,9	1,9%	-14,9%



### Growth rates: IPI raw and wda

Month	Raw		Wda		Working days		G_y raw	G_y wda
	2012	2013	2012	2013	2012	2013		
jan	89,2	89	88,9	86	21	22	-0,2%	-3,3%
feb	99	90,9	96,4	92,1	21	20	-8,2%	-4,5%
mar	105,3	94,8	104,9	98,8	22	21	-10,0%	-5,8%
apr	89,4	89	93,2	88,6	19	20	-0,4%	-4,9%
may	105,4	100,6	104,1	99,3	22	22	-4,6%	-4,6%
jun	99,5	94,2	99,1	96,8	21	20	-5,3%	-2,3%
jul	107,6	106,8	108,6	104,5	22	23	-0,7%	-3,8%
aug	62,2	57,5	61,4	58,6	22	21	-7,6%	-4,6%
sep	96,5	96,7	101,4	98,5	20	21	0,2%	-2,9%
oct	103,4	103	101,2	100,8	23	23	-0,4%	-0,4%
nov	96	93,9	95,6	96,5	21	20	-2,2%	0,9%
dec	78,4	79,9	81,3	80,3	19	20	1,9%	-1,2%

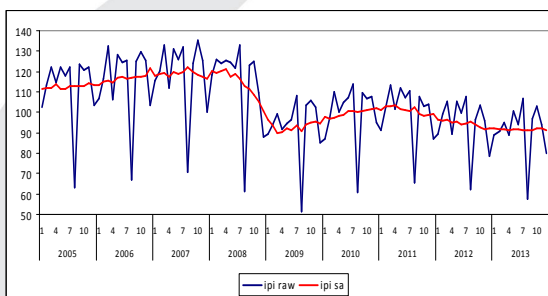


### Growth rates G\_m: IPI sa

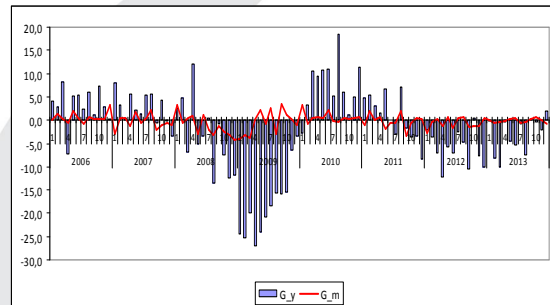
Month	SA		G_m
	2012	2013	
jan	96,4	92,3	0,1%
feb	96,1	91,8	-0,5%
mar	96,4	91,5	-0,3%
apr	95,1	91,3	-0,2%
may	95,7	91,5	0,2%
jun	94,2	91,9	0,4%
jul	94,7	91,4	-0,5%
aug	95,4	91,1	-0,3%
sep	94	91,4	0,3%
oct	92,8	92	0,7%
nov	91,7	92,1	0,1%
dec	92,2	91,4	-0,8%



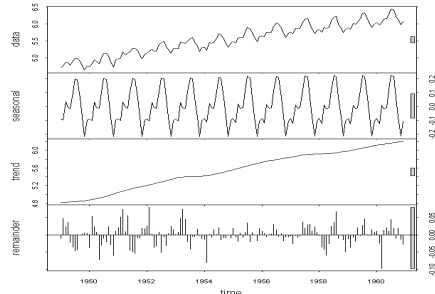
### IPI: raw and sa



### IPI: G\_m and G\_y



## Component: AirPassengers



## Components:

- Trend: T
- Cycle: C
- Seasonal: S
- Irregular: I

Additive components:

$$X_t = T_t + C_t + S_t + I_t$$

## Trend-Cycle

1. The trend is the long term evolution of the series that can be observed on several decades. Smooth component without sudden variations. Difficult to estimate due to the shortness of time series
2. The cycle is the smooth and recurrent movement that can usually be observed around the long term trend. Important for business cycle analysis

## Seasonal and Irregular

1. The seasonal component represents fluctuations observed during the year and which appears to repeat itself on a more or less regular basis from one year to other. Needed to be removed to interpret the short-term evolution.
2. The irregular is composed of residual and random fluctuations that cannot be assigned to the other components and considered as residual (capturing remaining short-term fluctuations).

## Cause of seasonality

1. Seasonality and climate: due to the variations of weather
2. Habits and practices: social habits or administrative rules (school holidays, summer closing, christmas shopping, end-of-season sales)

## Seasonal Adjustment

1. Process of estimating and removing seasonal effects from a time series that happen at the same time and with similar magnitude
2. Can be applied only if data are collected at frequency less than annually
3. Improvement of comparability over time and across space

### Calendar effects

1. During the process of seasonal adjustment, if present, we remove calendar effects as well
  - Trading day effect
  - Moving Holiday effect
  - Leap year



### Trading day effect

1. Recurrent effects associated with individual days of week
2. Some months have an excess of working days
3. Two different ways to model
  - Six variables counting the number of mondays, tuesdays, ecc
  - One variable considering the composition working days in the week



### Moving Holiday Effect

1. Effects from holidays that are not always on the same day of the month such as, for most of european countries, Easter. Easter moves between days but also can move between months since it can occur in March or April

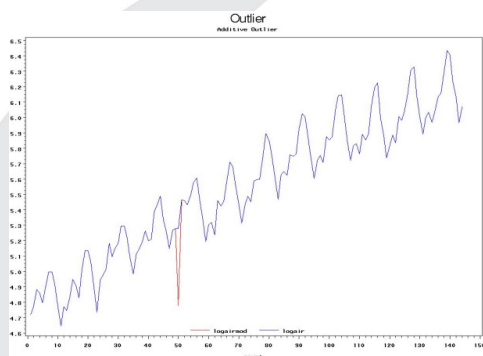


### Outlier

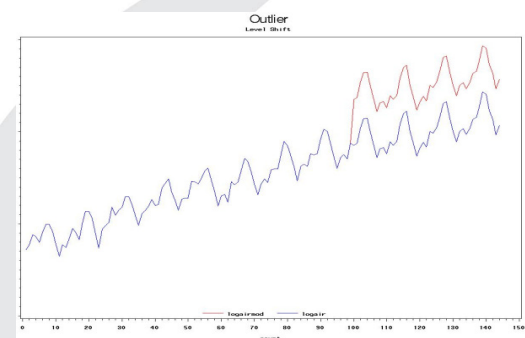
1. Further component
2. If there is some kind of unusual event the information is retained and kept. Outliers are included in the seasonally adjust series
  - Additive
  - Level Shift
  - Temporary change



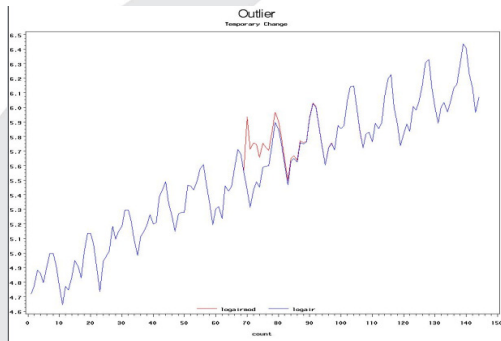
### Additive outlier



### Level shift



### Temporary change



### Part 4

### How to seasonally adjust? Background

1. Models for time series



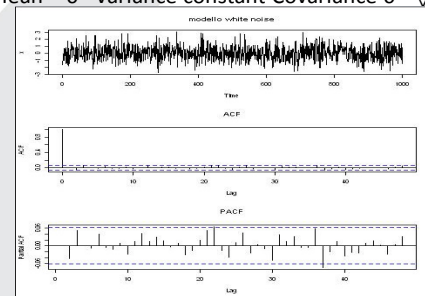
### Difference with classic statistics

1. A time series is a set of observations ordered in time. We will consider only time series observed at a discrete set of evenly spaced time intervals
2. A statistical time series can be considered as resulting from some underlying statistical (also called stochastic) process. The process might be represented as a mathematical model. The time series can be considered a single realization of the generating process.



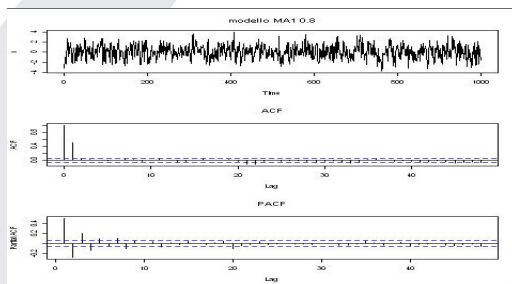
### White noise process

Mean = 0 Variance constant Covariance 0  $\forall k$



### Moving Averages Process of order 1

$$X_t = \varepsilon_t - \theta_1 \varepsilon_{t-1}$$



### MA(q) process

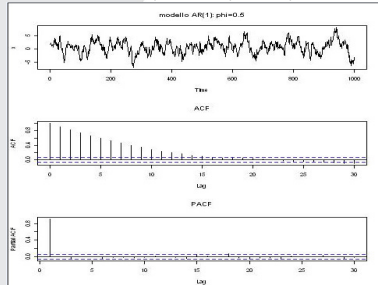
- The time series is regarded as a moving average (unevenly weighted) of a random shock series  $\varepsilon_t$

$\phi_1$



### Autoregressive Process of order 1

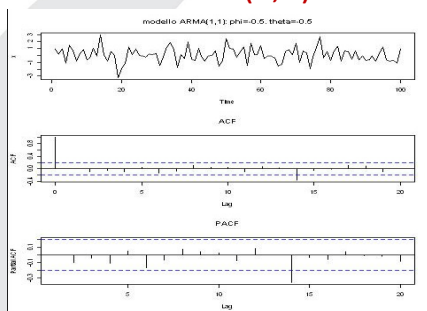
$$X_t = \phi_1 X_{t-1} + \varepsilon_t$$



### AR(p) process

1. An AR model expresses a time series as a linear function of its past values .
2. The order of the AR model tells how many lagged past values are included
3. AR(1) model has the form of a regression model in which  $X_t$  is regressed on its previous value. In this form  $\phi_1$  is analogous to the regression coefficient and  $\varepsilon_t$  the regression residuals.

### ARMA(1,1)



### ARMA(p,q) process

We have seen that the autoregressive model includes lagged terms on the time series itself, and that the moving average model includes lagged terms on the noise or residuals. By including both types of lagged terms, we arrive at what are called autoregressive –moving average or ARMA(p,q) models. The order of the ARMA model is included in parentheses as ARMA(p,q), where p is the autoregressive order and q the moving-average order.

### Model for non-stationary time series

#### ARIMA(p,d,q) process

AutoRegressive Integrated Moving Average of order p, d and q

$$(1 - \phi_1 B - \phi_2 B^2 - \dots - \phi_p B^p) (1 - B)^d X_t = (1 - \theta_1 B - \theta_2 B^2 - \dots - \theta_q B^q) a_t$$

$$\phi(B) (1 - B)^d X_t = \vartheta(B) a_t$$

1.  $\phi(B)$ : stationary  $\phi_1$
2.  $\vartheta(B)$ : invertible
3.  $(1 - B)^d$ : unit (nonstationary) roots

### Differencing

ARIMA models are defined for stationary time series. Therefore, if you start off with a non-stationary time series, you will first need to ‘difference’ the time series until you obtain a stationary time series.

If you have to difference the time series d times to obtain a stationary series, then you have an ARIMA(p,d,q) model, where d is the order of differencing used.

### Integration: random walk

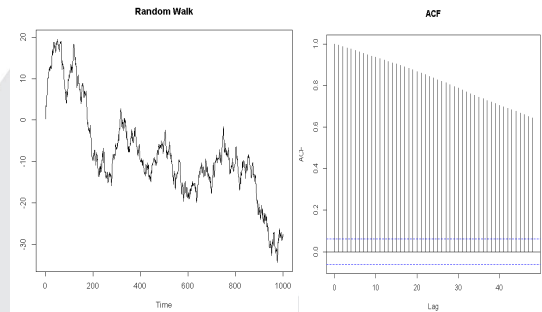
Two processes apparently similar:

- 1)  $X_t = \phi_1 X_{t-1} + \varepsilon_t$
- 2)  $X_t = X_{t-1} + \varepsilon_t (\phi_1 = 1)$

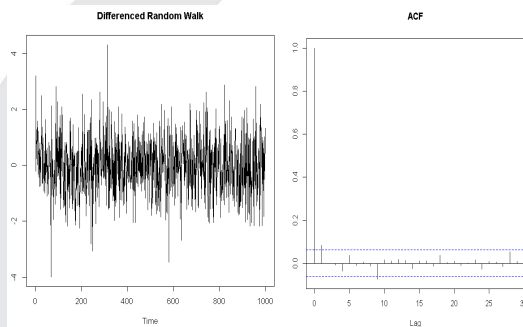
When we consider the statistical behavior of the two processes by investigating the mean (the first moment), and the variance and autocovariance (the second moment), they are completely different. Although the two process belong to the same AR(1) class: (1) is a stationary process, while (2) is a nonstationary process



### Random walk



### Differenced Random walk



### Arima model for seasonality (SARIMA)

$$ARIMA(p,d,q)(P,D,Q)_s$$

$$\varphi(B)(1 - \Phi_1 B^s - \Phi_2 B^{2s} - \dots - \Phi_p B^{ps}) (1-B)^d (1-B^s)^D X_t = \vartheta(B)(1 - \Theta_1 B^s - \Theta_2 B^{2s} - \dots - \Theta_q B^{qs}) a_t$$

$$\varphi(B)\Phi(B)(1-B)^d (1-B^s)^D X_t = \vartheta(B)\Theta(B) a_t$$

$\varphi(B)$  and  $\Phi(B)$ : stationary  
 $\vartheta(B)$  and  $\Theta(B)$ : invertible  
 $(1-B)^d$  and  $(1-B^s)^D$ : unit (nonstationary) roots



### RegArima

$$Y_t = \alpha + \sum_{j=1}^p \beta_j Z_{j,t} + X_t$$

Deterministic component  
 Stochastic ARIMA Model



### RegArima: deterministic effects

$$Y_t = \alpha + \sum_{j=1}^p \beta_j Z_{j,t} + X_t$$

- $\alpha$  Constant
- $Y_t$  Observed time series
- $Z_{j,t}$   $j$ -th regressor (Outlier, Trading Days, User)
- $\beta_j$   $j$ -th parameter
- $X_t$  Residuals (autocorrelated!!)



## Part 5 How to seasonally adjust?

1. Deterministic seasonality
2. Tramo-Seats and X-12-Arima

## Deterministic seasonality

**Using Dummy Variables in MRL to Account for Seasonality** A more sophisticated and convenient means of creating (additive) seasonality adjustments is to employ dummy variables in a multiple linear regression. That is, if there are  $k$  seasons, one can think of making up  $k - 1$  dummy variables  $x_1, x_2, \dots, x_{k-1}$  where for period  $t$

$$x_{j,t} = \begin{cases} 1 & \text{if period } t \text{ is from season } j \\ 0 & \text{otherwise} \end{cases}$$

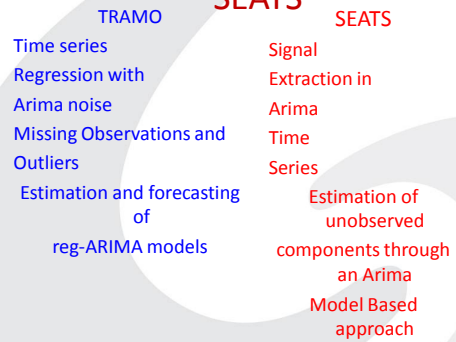
and then using these in a Multiple Linear Regression, fitting (for example)

$$y_t \approx b_0 + b_1 t + a_1 x_{1,t} + a_2 x_{2,t} + \dots + a_{k-1} x_{k-1,t}$$

## Deterministic seasonality: how to model

Period	JC Penney Sales	1st Quarter	2nd Quarter	3rd Quarter
1	4452	1	0	0
2	4507	0	1	0
3	5537	0	0	1
4	8157	0	0	0
5	9481	1	0	0
6	8420	0	1	0
7	7208	0	0	1
8	9559	0	0	0
9	8755	1	0	0
10	8483	0	1	0
11	7159	0	0	1
12	9072	0	0	0
13	7339	1	0	0
14	7194	0	1	0
15	7639	0	0	1
16	9661	0	0	0
17	7528	1	0	0
18	7207	0	1	0
19	7538	0	0	1
20	9673	0	0	0
21	7522	1	0	0
22	7211	0	1	0
23	7259	0	0	1
24	9542	0	0	0

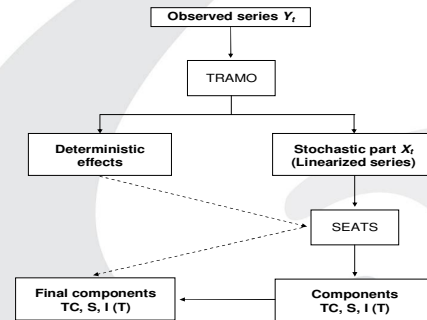
## Stochastic seasonality: TRAMO-SEATS



## TRAMO

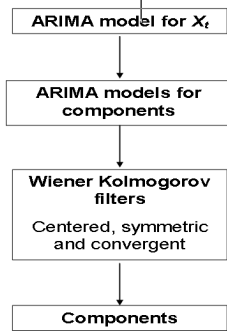
- performs a pretesting to decide between a log transformation and no transformation
- identifies the ARIMA model through an automatic model identification procedure
- pretests for the presence of trading-day, leap-year and Easter effects
- interpolates missing values
- detects outliers (AO, LS and TC)
- estimates the model
- computes forecasts

## TRAMO and SEATS





## TRAMO and SEATS: approach



## SEATS

- decomposes the ARIMA models (canonical decomposition)
- derives the WK filters
- estimates the components
- estimates the final components
- computes the forecasts for components and the respective standard error