Concepts and Terminology for International Comparisons of Prices and Real Incomes

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Motivation

- Use of Purchasing Power Parities is quite prevalent.
- PPPs are available from the International Comparison Program on a regular basis
  - ICP 2005 covered 146 countries and ICP 2011 will cover 180 countries
- Extrapolated PPPs covering 170 countries for the period 1950 to 2010 are available from Penn World Tables.
- PPPs and real GDP (and its components) over time and space are available from PWT and other sources.
  - Real GDP comparisons across countries at a given point of time (current price comparisons)
  - Real GDP comparisons at constant period $t$ comparisons
Motivation

- Despite the prevalence of use of PPPs, users are not familiar with the concepts used in international comparisons.
- There is no established notation or conceptual framework that is commonly used in ICP and PWT publications.
- There is confusion even among researchers who are involved in international comparisons.
  - PWT uses nomenclature that is not standard in ICP as PWT covers both space and time whereas ICP covers over space at a given period of time
- The main objective of our work is to streamline the concepts and discuss the relationships between concepts.
Key Concepts and Notation

Gross Domestic Product of country j
(in national currency units)

Nominal GDP: GDP of country i in period t expressed in reference currency units using exchange rates

Real GDP: GDP of country i expressed in reference currency units converted using PPPs
Purchasing Power Parities (PPPs)

PPPs are amounts of currencies, of different countries, that have the same purchasing power as one unit of a reference currency (e.g. US$) with respect to a selected basket of goods and services.
PPPs - basic uses

- PPPs are essentially spatial price index numbers – interregional comparisons
- PPPs are used for real income comparisons
  - World Bank Poverty measures using $1 and $2/day poverty lines
  - WDI and HDI; regional and global inequality
- Growth and convergence studies
  - Necessary to have panels of real incomes – nominal incomes adjusted for temporal and spatial price differences
- Growth and productivity studies
  - Computing TFP measures, decomposition to technical change and efficiency change
- PPPs are very useful for cross-country analyses by researchers, industries and international organisations.
Multilateral Comparisons

Price and Quantity Data

<table>
<thead>
<tr>
<th>Countries</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>i</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$p_{11}$</td>
<td>$q_{11}$</td>
<td>$p_{21}$</td>
<td>$q_{21}$</td>
<td>$p_{31}$</td>
</tr>
<tr>
<td>2</td>
<td>$p_{12}$</td>
<td>$q_{12}$</td>
<td>$p_{22}$</td>
<td>$q_{22}$</td>
<td>$p_{32}$</td>
</tr>
<tr>
<td>3</td>
<td>$p_{13}$</td>
<td>$q_{13}$</td>
<td>$p_{23}$</td>
<td>$q_{23}$</td>
<td>$p_{33}$</td>
</tr>
<tr>
<td>j</td>
<td></td>
<td></td>
<td></td>
<td>$p_{ij}$</td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>$p_{1I}$</td>
<td>$q_{1I}$</td>
<td>$p_{2I}$</td>
<td>$q_{2I}$</td>
<td>$p_{3I}$</td>
</tr>
</tbody>
</table>
Matrix of Multilateral Comparisons

\[ I_{MxM} = \begin{bmatrix}
I_{11} & I_{12} & \cdots & I_{1M} \\
I_{21} & I_{22} & \cdots & I_{2M} \\
\vdots & \vdots & \ddots & \vdots \\
I_{M1} & \cdots & \cdots & I_{MM}
\end{bmatrix} \]

Transitivity: \( I_{jk} = I_{jl} \times I_{lk} \) - a consistency requirement

Result: A matrix of multilateral comparisons satisfies transitivity if and only if there exist \( \pi_1, \pi_2, \ldots, \pi_I \) such that

\[
I_{jk} = \frac{\pi^k}{\pi^j} \Rightarrow PPP^1 = 1; \ PPP^2 = \pi^2; \ldots PPP^I = \pi^I
\]
Notes on PPPs

- Normalisation used in computing PPPs is quite important.
- PPPs have two components: *price level* and *currency unit* components.
- PPPs cannot be directly interpreted as *spatial price index numbers* unless
  - The currency units in the countries compared are the same; or
  - PPPs are computed using price data that is converted into a common currency using exchange rates.
- Concept of *price level* is used within the ICP.
- *Real GDP* for country $i$ is $\text{RGDP}^i = \frac{\text{GDP}^i}{\text{PPP}^{1i}}$.
## PPPs from ICP 2005

<table>
<thead>
<tr>
<th>Country</th>
<th>Exch. Rate</th>
<th>PPP</th>
<th>PLI% (US=100)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P.R. China</td>
<td>8.19</td>
<td>3.56</td>
<td>43</td>
</tr>
<tr>
<td>Hong Kong</td>
<td>7.78</td>
<td>5.86</td>
<td>75</td>
</tr>
<tr>
<td>India</td>
<td>44.10</td>
<td>15.15</td>
<td>34</td>
</tr>
<tr>
<td>Australia</td>
<td>1.31</td>
<td>1.39</td>
<td>106</td>
</tr>
<tr>
<td>Japan</td>
<td>110.22</td>
<td>129.55</td>
<td>118</td>
</tr>
<tr>
<td>Switzerland</td>
<td>1.25</td>
<td>1.75</td>
<td>140</td>
</tr>
<tr>
<td>Ethiopia</td>
<td>8.67</td>
<td>2.33</td>
<td>27</td>
</tr>
</tbody>
</table>

Price Level Indices

Price Level Index (PLI): It is defined as the ratio of PPP to the exchange rate

\[ PLI^{1j} = \frac{PPP^{1j}}{XR^{1j}} \]

Properties of PLIs:
- By definition PLI of reference country is 1
- Interpretation of PLI can be ambiguous
- PLIs for a country using two different reference countries are not directly comparable
- PLIs are transitive
Price Level Indices

Price Level Index (PLI): It is defined as the ratio of PPP to the exchange rate

\[ PLI_{1j} = \frac{PPP_{1j}}{XR_{1j}} = \frac{GDP_j}{XR_{1j}} = \frac{GDP_j}{PPP_{1j}} \]

Properties of PLIs:
- By definition PLI of reference country is 1. For this country the nominal and real GDP are the same
- Interpretation of PLI can be ambiguous
- PLIs for a country using two different reference countries are not directly comparable
- PLIs are transitive
Price Level Indices Relative to World Average

1. A simple approach is to compute the geometric average of all PLI’s and normalise PLI of each country using the geometric mean.
   - Such a normalisation does not have any useful interpretation.

2. An alternative is to compute the weighted average of PLIs and use a normalisation based on the average.

   \[
   \text{World average of } PLIs = \mu^1 = \frac{\sum_{j=1}^{I} PLI^{1j} \cdot RGDP^{1j}}{\sum_{j=1}^{I} RGDP^{1j}}
   \]

3. Scale the PPPs to the world average as:

   \[
   PPP^{wj} = PPP^{1j} / \mu^1
   \]
Price Level Indices Relative to World Average

4. PLI for country \( j \) with world average is:

\[
PLI^{W_j} = \frac{PPP^{W_j}}{XR^{1j}}
\]

5. World average of PLI’s at new PPPs is equal to 1 and, therefore:

\[
\sum_{j=1}^{I} \frac{GDP^j}{PPP^{W_j}} = \sum_{j=1}^{I} \frac{GDP^j}{XR^{1j}}
\]

World GDP at the new PPPs is equal to world nominal GDP at exchange rate

6. PLI’s defined using world average are invariant to the choice of the reference country for PPPs.
Table 1: Price Level Indexes for Selected Countries, 2005
(Price level in USA = 1.0)

<table>
<thead>
<tr>
<th>Country</th>
<th>PPP</th>
<th>XR</th>
<th>PLI with USA = 1</th>
<th>PLI with world average. =1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>1.39</td>
<td>1.31</td>
<td>1.06</td>
<td>1.32</td>
</tr>
<tr>
<td>Denmark</td>
<td>8.52</td>
<td>5.95</td>
<td>1.42</td>
<td>1.76</td>
</tr>
<tr>
<td>India</td>
<td>14.67</td>
<td>44.10</td>
<td>0.33</td>
<td>0.41</td>
</tr>
<tr>
<td>Brazil</td>
<td>1.36</td>
<td>2.43</td>
<td>0.56</td>
<td>0.69</td>
</tr>
<tr>
<td>South Africa</td>
<td>3.87</td>
<td>6.36</td>
<td>0.61</td>
<td>0.76</td>
</tr>
<tr>
<td>USA</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1.24</td>
</tr>
</tbody>
</table>
Real GDP and Constant Price Comparisons

Real GDP: GDP of country j expressed in reference currency units converted using PPPs

$$RGDP_t^i = \frac{GDP_t^i}{PPP_t^i}$$

Real GDP is comparable across countries as the aggregate is adjusted for price level differences across countries.

Constant Price Comparisons:

Let $$\pi_n$$ (n=1,2,...,N) be a set of commodity prices in the currency units of a reference country. Let CGDP represent GDP at constant prices. Then

$$CGDP^j = \sum_{n=1}^{N} \pi_n x_n^j$$
1. RGDP and CGDP are used interchangeably though they are conceptually different.

2. For any given CGDP comparison with a given set of prices $\pi$, it is possible to define PPPs as:

$$\text{PPP}_j = \frac{\sum_{n=1}^{N} p_n^j \cdot x_n^j}{\sum_{n=1}^{N} \pi_n \cdot x_n^j} = \frac{\text{GDP}_j}{\sum_{n=1}^{N} \pi_n \cdot x_n^j}$$

which implies

$$\text{CGDP}_j = \sum_{n=1}^{N} \pi_n \cdot x_n^j = \frac{\text{GDP}_j}{\text{PPP}_j} = \text{RGDP}_j$$

3. But the converse is not true. For example, if PPPs are obtained used GEKS, we cannot find a corresponding vector of prices $\pi$ that can define CGDP.
4. The vector of prices $\pi$ are referred to as *international prices*.

5. Geary-Khamis, Iklé-Dikhanov-Balk and Rao methods provide alternative sets of *international prices*.

6. GK and IDB methods use linear functions to define PPPs associated with a given set of international prices.


8. Feenstra-Ma-Rao use Neary approach to derive constant price real income comparisons.
Table 2: PPPs and Real Incomes for Selected Asia-Pacific Countries, 2005 and 2009

Reference Currency – Hong Kong dollar

<table>
<thead>
<tr>
<th>Country</th>
<th>2005</th>
<th></th>
<th>Real per capita GDP</th>
<th>2009</th>
<th></th>
<th>Real per capita GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PPP</td>
<td>XR</td>
<td>PLI*</td>
<td>PPP</td>
<td>XR</td>
<td>PLI*</td>
</tr>
<tr>
<td>Bangladesh</td>
<td>3.98</td>
<td>8.27</td>
<td>86</td>
<td>4.16</td>
<td>8.91</td>
<td>76</td>
</tr>
<tr>
<td>China, PR</td>
<td>0.61</td>
<td>1.05</td>
<td>103</td>
<td>0.64</td>
<td>0.88</td>
<td>117</td>
</tr>
<tr>
<td>Hong Kong</td>
<td>1</td>
<td>1</td>
<td>180</td>
<td>1</td>
<td>1</td>
<td>162</td>
</tr>
<tr>
<td>India</td>
<td>2.58</td>
<td>5.67</td>
<td>82</td>
<td>2.70</td>
<td>6.25</td>
<td>70</td>
</tr>
<tr>
<td>Malaysia</td>
<td>0.31</td>
<td>0.49</td>
<td>112</td>
<td>0.29</td>
<td>0.45</td>
<td>104</td>
</tr>
<tr>
<td>Thailand</td>
<td>2.80</td>
<td>5.17</td>
<td>97</td>
<td>2.47</td>
<td>4.42</td>
<td>91</td>
</tr>
<tr>
<td>Vietnam</td>
<td>829</td>
<td>2039.12</td>
<td>73</td>
<td>950.68</td>
<td>2201.95</td>
<td>70</td>
</tr>
</tbody>
</table>

Note: (*) PLIs are defined relative to regional average with Asia = 100.
Source: ADB (2007) and ADB (2012)
1. These are comparisons across countries at a given point of time using PPPs for that period.

\[ RGDP^j_t = \frac{GDP^j_t}{PPP^j_t} \]

- \( RGDP^j_t \) are comparable across countries in a given year.
- These are typical outputs of ICP in a given benchmark
- \( RGDP^j_t \) is not directly comparable to \( RGDP^j_s \) for \( t \neq s \).
- These series are referred to as Real GDP series at current prices.
1. These are GDP figures converted into the currency of a reference country $j$ and a reference/base period $t$.

2. Let $PPP_{ts}^{jk}$ represent PPP for country $k$ in period $s$ relative to a reference country $j$ in reference period $t$, then

$$CRGDP_{ts}^{jk} = \frac{GDP_{s}^{k}}{PPP_{ts}^{jk}}$$


3. PWT refers to these series as *Real GDP series at constant year* $t$ *prices*.

4. Construction of *constant price* series is still being researched.
Thank you!