Weights in CPI/HICP and in seasonally adjusted series

Jan Walschots

Ottawa Group Meeting
Copenhagen, 2 May 2013
Subject of the paper

- Alternative ways to write the Laspeyres index formula and to calculate weights.
- Annual baskets and monthly indices.
- The impact of seasonal adjustment on the weights.
- Theoretical results applied to ECB series of the HICP.
LASPEYRES index

Basic formula:

\[ U_{ab} = \frac{\sum_i P_{ib} \times Q_{ia}}{\sum_i P_{ia} \times Q_{ia}} \]

Index, written as elementary aggregates and weights:

\[ U_{ab} = \frac{\sum_i P_{ib} \times Q_{ia}}{\sum_i P_{ia} \times Q_{ia}} = \frac{\sum_i P_{ib} / P_{ia} \times P_{ia} \times Q_{ia}}{\sum_i P_{ia} \times Q_{ia}} = \sum_i \frac{P_{ia} \times Q_{ia}}{P_{ia}} \times \frac{P_{ib}}{P_{ia}} = \sum_i W_{ia} \times \frac{P_{ib}}{P_{ia}} \]
**Fixed basket, written as chain index**

\[
U_{ab} = \frac{\sum_i P_{ib} \cdot Q_{ia}}{\sum_i P_{ia} \cdot Q_{ia}} = \frac{\sum_i P_{i(a+1)} \cdot Q_{ia}}{\sum_i P_{ia} \cdot Q_{ia}} \cdot \frac{\sum_i P_{i(a+2)} \cdot Q_{ia}}{\sum_i P_{i(a+1)} \cdot Q_{ia}} \cdot \ldots \cdot \frac{\sum_i P_{ib} \cdot Q_{ia}}{\sum_i P_{i(b-1)} \cdot Q_{ia}}
\]

\[
U_{ab} = \prod_{x=a}^{b-1} \left( \frac{\sum_i P_{i,x+1} \cdot Q_{ia}}{\sum_i P_{ix} \cdot Q_{ia}} \right) = \prod_{x=a}^{b-1} \left( \sum_i W_{i,a,x} \cdot \frac{P_{i,x+1}}{P_{ix}} \right)
\]

\[
W_{i,a,x} = \frac{P_{ix} \cdot Q_{ia}}{\sum_i P_{ix} \cdot Q_{ia}} = W_{i,a,a} \cdot \frac{P_{ix}}{P_{ia}} \cdot \frac{\sum_i P_{ia} \cdot Q_{ia}}{\sum_i P_{ix} \cdot Q_{ia}}
\]
Real chain index, with changing basket

(7) \[ U_{ab} = \prod_{x=a}^{b-1} \left( \sum_i W_{ix} \left( \frac{P_{i,x+1}}{P_{ix}} \right) \right) \]

Weights in HICP are December weights

(10) \[ W_{i(D,A)} = \frac{P_{i(D,A)} * Q_{i(D,A)}}{\sum_i P_{i(D,A)} * Q_{i(D,A)}} \]

And are based on an annual basket

(12) \[ W_{i(D,A)} = \frac{P_{i(D,A)} * Q_{iA}}{\sum_i P_{i(D,A)} * Q_{iA}} \]
Annual baskets and monthly indices.

- Monthly figures may lead to seasonal patterns in consumption and in prices.
- Seasonal pattern in consumption is ruled out of HICP by taking annual baskets and by the Regulation on seasonal products.
- In HICP series, using Laspeyres type index, there is only a seasonal pattern in prices.
How to deal with seasonal pattern of price indices

• Focus on annual rates
• Seasonal adjustment of index series
  • Not performed by CBS, Eurostat and many Statistical institutes
  • Often performed by National Banks, research institutes and European Central Bank (ECB)
Direct and indirect seasonal adjustment

- Direct approach: SA applied to headline series. (outside scope of this paper)
- Indirect approach: SA applied to components of HICP and aggregated to total SA series
- Components may have stable and identifiable patterns where total pattern changes due to changing weights
Original and SA weights in fixed base Laspeyres index

- Original series

\[
W_{ia} = \frac{P_{ia} * Q_{ia}}{\sum_i P_{ia} * Q_{ia}}
\]

(15)

- Seasonally adjusted series

\[
W_{ia}^{sa} = \frac{P_{ia}^{sa} * Q_{ia}^{sa}}{\sum_i P_{ia}^{sa} * Q_{ia}^{sa}}
\]

(17)
Original and SA weights in fixed base Laspeyres index

Considering that $Q_{ia}^{sa} = Q_{ia}$, combining (15) and (17) leads to:

\[
W_{ia}^{sa} = W_{ia} \times \frac{P_{ia}^{sa}}{P_{ia}} \times \frac{\sum P_{ia} \times Q_{ia}}{\sum P_{ia}^{sa} \times Q_{ia}}
\]

(19)

\[
W_{ia}^{sa} \cong W_{ia}
\]
Original and SA weights in chain-linked Laspeyres index (HICP)

(22) \[ W_{i(D,A)}^{sa} = \frac{P_{i(D,A)}^{sa} \cdot Q_{iA}^{sa}}{\sum_i P_{i(D,A)}^{sa} \cdot Q_{iA}^{sa}} \]

(23) \[ W_{i(D,A)}^{sa} = W_{i(D,A)} \cdot \frac{P_{i(D,A)}^{sa} \cdot Q_{iA}^{sa}}{P_{i(D,A)} \cdot Q_{iA}} \cdot \frac{\sum_i P_{i(D,A)} \cdot Q_{iA}^{sa}}{\sum_i P_{i(D,A)}^{sa} \cdot Q_{iA}^{sa}} \]
Original and SA weights in chain-linked Laspeyres index (HICP)

The last term of (23) is a constant and the quantities $Q_{ia}$ are not seasonally adjusted. Therefore:

\[ W_{i(D,A)}^{sa} \propto W_{i(D,A)} \cdot \frac{P_{i(D,A)}^{sa}}{P_{i(D,A)}} \]  \hspace{2cm} (24)

with

\[ \sum_{i} W_{i(D,A)}^{sa} = 1 \]  \hspace{2cm} (25)
Incomplete availability of data

- December indices from original and SA series are published and computed from the weights of year A-1
- Annual weights for year A are usually not published
- Formulas (24/25) are based on expenditures in year A and can not be calculated exactly.
Current practice of ECB

ECB calculates SA series for:

- Food
- Industrial goods excluding energy
- Services

and aggregates with original series for

- Energy

to derive SA headline series
Aggregation of ECB SA series using adjusted weights

- SA series for euro area. Source ECB
- December Weights. Source Eurostat
- December indices. Source Eurostat
- SA December indices. Source ECB

give adjusted weights for aggregation of SA series as in (24)
Results SA HICP aggregated series euro area

Impact of adjusted weights depends on:
• Distance of December prices from trend
• Importance of seasonal pattern for product groups that have trends that differ from headline HICP
• Level of aggregation of index components
• Impact for individual countries maybe bigger than for euro area.
Difference between SA series with adjusted weights and published SA series, HICP euro area
Conclusions

• On theoretical considerations weights for SA series should be adjusted in the case of a chain-linked Laspeyres index
• In practice the impacts may be very small
• Not all data for adjusted weights are published by Eurostat and NSIs