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## A note on lags and leads in the fiscal reaction function for ADAM

### Abstract:

*In this note we consider a public purchase experiment and compare how the reaction in income tax rate and labor market depends on the specification of the fiscal reaction function for ADAM. The fiscal rule should be both backward and forward looking in the income tax rate, which is the instrument, and the rule should depend on current rather than lagged primary surplus and unemployment gap, which are the two goal variables. Without lagged and lagged tax rate, the reaction function seems to produce a volatile tax rate without achieving a lot of stability in the labor market reaction. We have seen in previous papers on the fiscal reaction function that we avoid unstable pro-cyclical fiscal policy by introducing leads and we can add that the lagged tax rate may help to optimize the fiscal reaction function. The lags and the leads minimize the over and undershooting of the fiscal instrument and make the changes in the instrument more unidirectional. This is not a new idea. It echoes e.g. Holbrook (1972), who concluded that when an instrument works with a lag, and most instruments work with a lag in ADAM, you should be patient and await the full result of the instrument change. Seen over a time period and not just looking at the first year, it is a bad idea to start with big instrument changes in an attempt to close the budget gap or the unemployment gap within the first year.*

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**Keywords:** fiscal reaction function, income tax rate, unemployment rate

*Modelgruppepapirer er interne arbejdsrapporter. De konklusioner, der drages i papirerne, er ikke endelige og kan være ændret inden opstillingen af nye modelversioner. Det henstilles derfor, at der kun citeres fra modelgruppepapirerne efter aftale med Danmarks Statistik.*

## Introduction

The fiscal reaction function for ADAM determines the income tax rate as an average of lagged and forward income tax rates, and the two other variables in the function are contemporaneous public primary surplus and contemporaneous unemployment gap. In this note we consider the public purchase experiment and assess how the formulation of lags and leads affects the response of the income tax rate and labor market. An optimal fiscal rule should produce a constant future tax rate as argued by Johnson (2001). Thus, we do not want volatility in the tax rate *ceteris paribus*, but we are also interested in reducing the volatility on the labor market, which is why the unemployment gap is included in the reaction function and obviously there is a potential trade-off between instrument and labor market volatility.

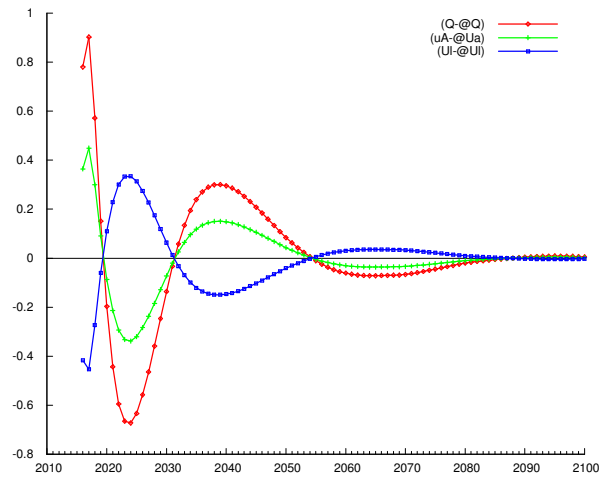
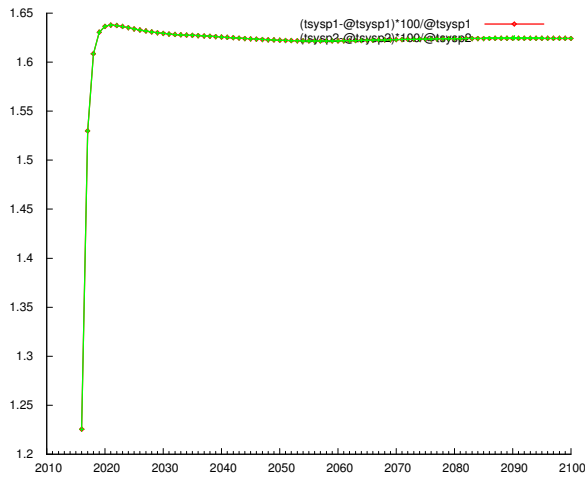
In the following, we test some alternatives to the fiscal reaction function for ADAM presented in working paper 'Fiscal Reaction function for ADAM targeting primary surplus and unemployment gap' and compare how the income tax rate and labor market reacts to higher public purchase shock. These changes include reducing the weight for the lagged tax rate or reducing the number of leads for the tax rates, and we also try to lag and lead the primary surplus and unemployment gap. For the first experiment, we use the reaction function presented in the working paper just mentioned.

*Case 0: the fiscal reaction function for ADAM targeting primary surplus and unemployment gap*

$$tsysp1 = \frac{\left( 5 * tsysp1_{-1} + \sum_{i=1}^{i=15} tsysp1_i \right)}{20} - ctsysp1 \left[ \frac{(Tfn\_o - (Tin\_o - Tirn\_o))}{Y} - bt \right] - cu \left[ (bulb - bulbw) - ut \right]$$

Where  $ctsysp1 = cu = 0.05$

The figures below present the response of income tax rate and labor market to one percent higher public purchase experiment using this fiscal reaction function. Compared to Johnson (2001), who originally inspired our reaction function, we are ok in so far as we do get a constant tax rate in steady state and the calculation is easy when using the base line developed for calculations with reaction function. Johnson has to split up the sample and make the long-run calculations with different versions of the IMF model, but with the proper base line we have no problem finding a steady state with constant tax rate and sustainable public finances in ADAM. The level of the debt ratio is changed by the fiscal shock but the ratio is constant in the long run both in the baseline and in the resulting after shock scenarios. The public primary surplus is zero in the long run both in the base line and in the resulting scenario, and the assumption of zero growth-corrected interest rate makes the public debt ratio constant.



*Case 1: Lagged primary surplus and lagged unemployment gap*

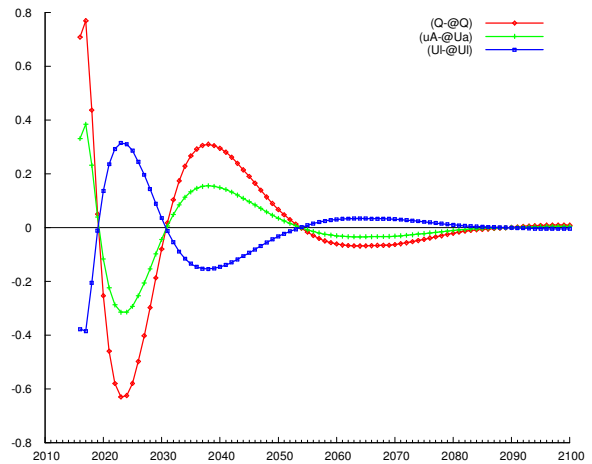
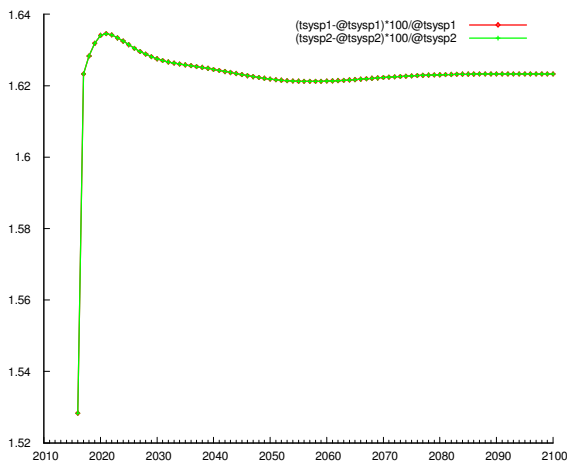
$$tsysp1 = \frac{\left( 5 * tsysp1_{-1} + \sum_{i=1}^{i=15} tsysp1_i \right)}{20} - ctsysp1 \left[ \frac{\left( Tfn\_o(-1) - (Tin\_o(-1) - Tim\_o(-1)) \right)}{Y(-1)} - bt(-1) \right] - cu \left[ (bulb(-1) - bulbw(-1)) - ut(-1) \right]$$

We introduce a minor change in the reaction function by lagging the public surplus and the unemployment gap. This change does not seem to matter. The response of income tax rate and labor market is the same as in the graph presented above under case 0.

*Case 2: only one lagged tsysp1*

When we reduce the weight for the first lag of income tax rate in the fiscal reaction function from  $\frac{5}{20}$  to  $\frac{1}{16}$ , the income tax rate overshoots a little more clearly in the short run and the rate undershoots a tiny bit as well on its path to steady state. Comparing case 2 to case 0 reveals that a higher income tax rate in the first few years dampens the immediate expensive labor market reaction a little bit. Thus, if the volatility of the tax rate is acceptable we may dampen the labor market volatility by reducing the weight for the lagged tax rate. However, the possible gain looks rather small.

$$tsysp1 = \frac{\left( tsysp1_{-1} + \sum_{i=1}^{i=15} tsysp1_i \right)}{16} - ctsysp1 \left[ \frac{\left( Tfn\_o - (Tin\_o - Tirm\_o) \right)}{Y} - bt \right] - cu \left[ (bulb - bulbw) - ut \right]$$

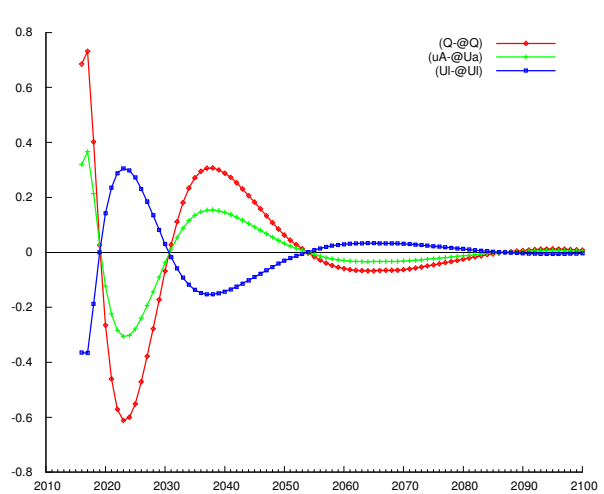
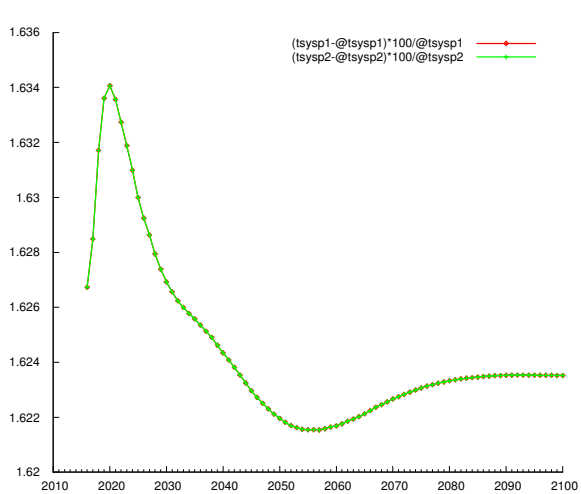


It can be added that if we use this fiscal reaction function with one period lagged primary surplus and lagged unemployment gap, we obtain the same response of income tax rate and labor market.

*Case 3: no lagged tsysp1*

$$tsysp1 = \frac{\left(\sum_{i=1}^{i=15} tsysp1_i\right)}{15} - ctsysp1 \left[ \frac{(Tfn\_o - (Tin\_o - Tirn\_o))}{Y} - bt \right] - cu [(bulb - bulbw) - ut]$$

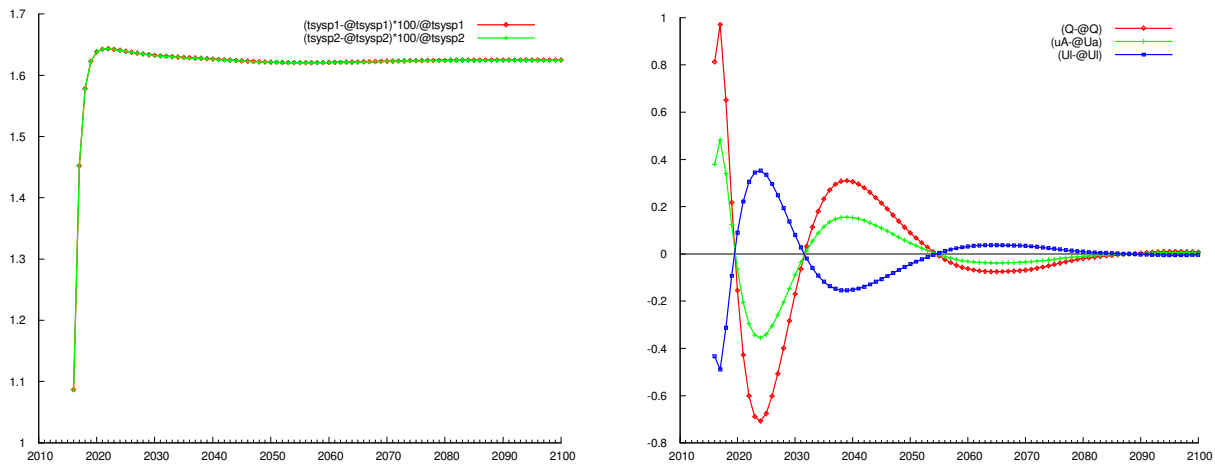
When the lagged income tax rate disappears from the fiscal reaction function, the response of the income tax rate becomes clearly more volatile and the rate does not find its steady state solution until the last decade of the simulation period.



*Case 4: only 10 leads in tsysp1*

In this calculation, we keep the original lag and its weight in the income tax rate but reduce the number of leads to ten. This makes the first year response of income tax rate a little lower and it increases the volatility in the labor market in the first few years. Apart from this, the long run solution remains stable and the labor market returns to the baseline. Thus, this specification can work in ADAM and it may be of interest depending on how the fiscal authority wants to change the income tax rate in the first year.

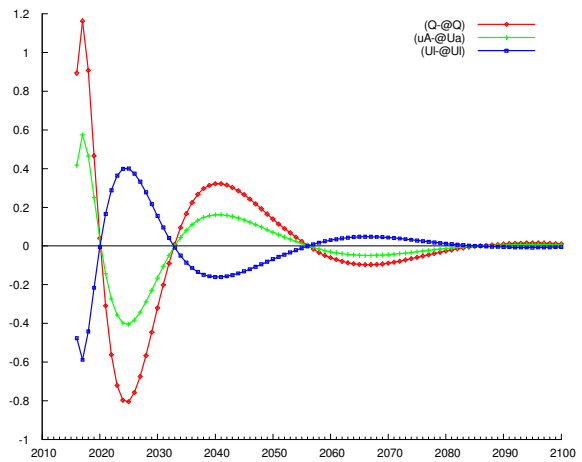
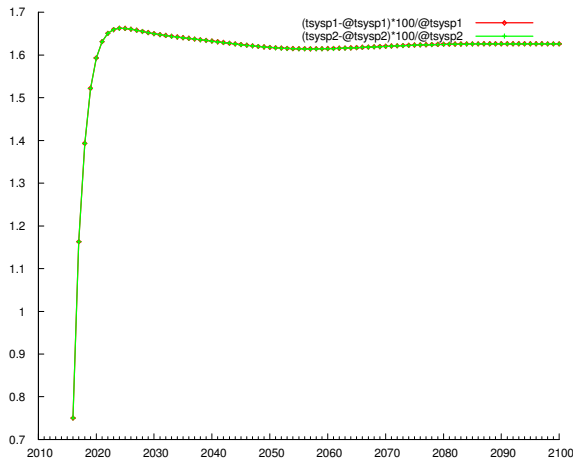
$$tsysp1 = \frac{\left( 5 * tsysp1_{-1} + \sum_{i=1}^{i=10} tsysp1_i \right)}{15} - ctsysp1 \left[ \frac{(Tfn\_o - (Tin\_o - Tirn\_o))}{Y} - bt \right] - cu \left[ (bulb - bulbw) - ut \right]$$



*Case 5: only 5 leads in tsysp1*

Now the tax rate is even less forward looking. The first-year response of income tax rate becomes smaller than in case 4 and consequently the immediate expansion on the labor market is allowed to be bigger.

$$tsysp1 = \frac{\left( 5 * tsysp1_{-1} + \sum_{i=1}^{i=5} tsysp1_i \right)}{10} - ctsysp1 \left[ \frac{(Tfn\_o - (Tin\_o - Tirn\_o))}{Y} - bt \right] - cu \left[ (bulb - bulbw) - ut \right]$$



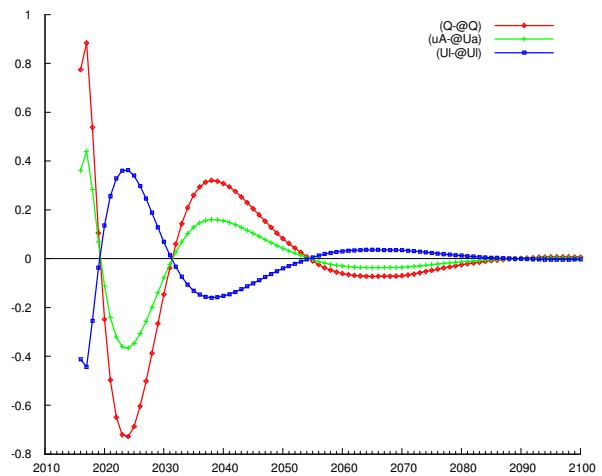
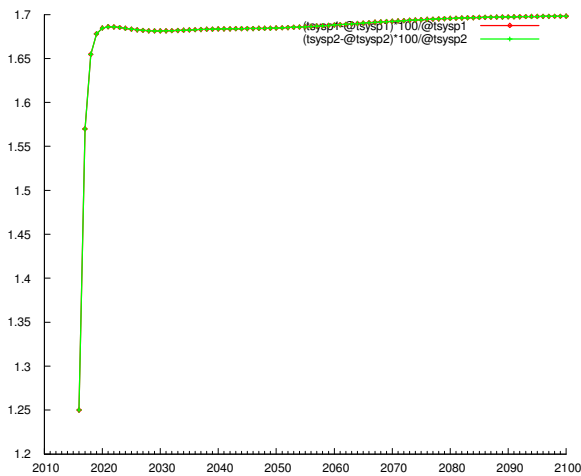
*Case 6: leaded looking primary surplus and leaded unemployment gap*

We now keep the original lags and leads in the tax rate but include a lead in both the public surplus and unemployment gap. Thus, both targets are forward looking, and this seems to make the reaction of the income tax rate slightly upward drifting over the long run. The first year response in the income tax rate is slightly different from case 0, but the clearest difference is the mentioned upward drift in the tax rate. The drift indicates that we are not finding steady state, and this may reflect the solution algorithm in Gekko.

$$tsysp1 = \frac{5 * tsysp1_{-1} + \sum_{i=1}^{i=15} tsysp1_i}{20} - (btact(+1) - btlead) - cu*(utact(+1) - utlead)$$

btact = (tfn\_o - (Tin\_o - Tirn\_o)) / y

utact = bulb - bulbw



## Conclusion

This note makes a public purchase experiment for different specifications of the fiscal reaction function, and we see that the response of the income tax rate and labor market depends on how the fiscal reaction function is formulated. More specifically, the simulations shows that if we make the fiscal rule less forward looking in the income tax rate, we get a lower first year response of income tax rate and more volatility in the labor market for a public purchase shock. On the other hand, if we make the fiscal rule somewhat less backward looking by reducing the weight for the lagged tax rate, the income tax rate overshoots a little more in short run and this short-run overshooting in the income tax rate can dampen the volatility in the labor market. However, if we simply skip the lagged tax rate we get significantly more volatility in the tax rate without dampening the labor market volatility. In the long run, the labor market response returns to the baseline for all cases presented in this note.

Summing up, this sensitivity analysis has shown that the fiscal rule should be both forward and backward looking.

This paper is the last in a series of papers on a fiscal reaction function for ADAM, and the whole list of papers is included in the listed references below. The first paper was made in November 2014.

All papers have one way or the other experimented with the specification of a fiscal policy function for ADAM. Maybe it is particularly difficult to specify a fiscal reaction function for ADAM. For instance, in a well-known international model like NIGEM, the reaction function looks much simpler. There are no leads and no unemployment gap. NIGEM's reaction function simply changes the income tax rate if the contemporaneous public budget deviates from its target. However, NIGEM has a monetary reaction function making the short-term interest rate a function of a GDP gap and there are leads in NIGEM's consumption function, which speeds up the reaction to tax changes. In other words, they do have leads and they do target the activity gap in NIGEM, but not in the fiscal reaction function.

Our argument for having the unemployment gap and not least the forward looking structure in the fiscal reaction function is that we assume that fiscal policy is formulated using ADAM. Besides, the Danish fixed exchange rate policy leaves no scope for a standard monetary policy rule, and we are uncertain about the empirical evidence for forward looking behavioral equations.

We have probably not found the best reaction function for ADAM but maybe it is good enough. It is always possible to experiment more with the specification of the reaction function but it may be helpful at this stage to try to generalize some of the arguments a little and think of the whole model.

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NIGEM (National Institute Global Economic Model)

<http://nimodel.niesr.ac.uk/nigem-intro/nigemintro.php?t=1&b=2&w=11>