Ordinal Utility & Cost of Living Indexes: How History Can Inform Contemporary Debates

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**Core Thesis:** The history of price index theory can clarify conceptual issues that still hinder contemporary discussions.
Key Arguments:

1. Despite using similar terms (e.g., utility), neoclassical economists have defined cost-of-living indexes (COLI’s) in quite different ways since the late 19th century.
2. The different definitions for COLI’s are tied in part to shifting ideas about utility, especially the change from cardinal to ordinal utility.
3. Each definition has a distinct way of handling changes in the utility functions of consumers.
4. How each form of COLI treats changes in utility functions affects its suitability for assessing changes in consumer welfare or adjusting benefit payments.
5. The most common definition today – e.g., Pollak (1989) – is not suitable for assessing changes in consumer welfare across two time periods, although it is frequently interpreted in that way.
What change in expenditure would be necessary to make a consumer equally well off in two situations (e.g., time period 1 and time period 2)?

– “well off” defined as psychological satisfaction or happiness

Given a consumer with a fixed utility function, what change in expenditure would be necessary to make that consumer indifferent between facing two sets of prices (e.g., from period 1 and from period 2)?
Let $\mathbf{X}$ be a vector of $n$ dimensions representing the quantities of goods and services in the set $x_1 \ldots x_n$.

An **ordinal utility function**, $U_O$, characterizes **preference rankings** for a particular consumer.

Given a collection of goods, $\mathbf{X}$, the function will define an indifference curve that contains other combinations of $x_1 \ldots x_n$ (other vectors, $\mathbf{X}$) among which the consumer is indifferent.

The full set of indifference curves defined by the utility function are ranked in order of preference.

Let $\mu_{o,X}$ be the ordinal ranking for the indifference curve of collection $\mathbf{X}$ in ordinal utility function $U_O$. 
A cardinal utility function, \( U_C \) characterizes the psychological satisfaction of a consumer.

Given a collection of goods, \( X \), the function will define the level of satisfaction, \( \mu_c \), produced by those goods.

Let \( \mu_{c,X} \) be the cardinal utility for the indifference curve of collection \( X \) in ordinal utility function \( U_O \).

In general, a cardinal utility function is dependent on both the underlying ordinal utility function and a set of exogenous factors, \( E \), such as mental and physical health, other aspects of the environment, etc.:

\[
\mu_{c,X} = U_C [U_O (X); E]
\]

Consequently, the value for \( \mu_{c,X} \) (cardinal utility) can change even while \( \mu_{o,X} \) (ordinal utility) remains constant.
<table>
<thead>
<tr>
<th>Type of COLI</th>
<th>Measures change in expenditures needed to...</th>
<th>Inter-temporal comparisons of welfare?</th>
<th>Changes in utility functions create...</th>
<th>Exemplars</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cardinal</strong></td>
<td>Maintain same level of cardinal utility, $\mu_c$</td>
<td>Yes</td>
<td>Operational problems</td>
<td>Edgeworth (1887-89) Pigou (1912)</td>
</tr>
<tr>
<td><strong>Weak Ordinal</strong></td>
<td>Remain on same indifference curve</td>
<td>Yes</td>
<td>Theoretical problems</td>
<td>Allen (1933) Hicks (1940)</td>
</tr>
<tr>
<td><strong>Strong Ordinal</strong></td>
<td>Remain on same indifference curve</td>
<td>No</td>
<td>No fundamental obstacles</td>
<td>Fisher &amp; Shell (1968) Pollak (1989)</td>
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</table>
Cardinal COLI:

Minimum change in expenditure needed to maintain the same level of cardinal utility in two time periods or locations

Method:

Assuming a constant utility function, use different market baskets (e.g., Laspeyres, Paasche) or combinations of indexes to bound the true index. **No clear solution in cases of changing utility function.**

“[Economic welfare is] the *psychic return of satisfaction,*” arising from “those parts of the community’s net income that enter easily into relation with the measuring rod of money.”

“[Our goal is] to devise, if we can, some measure of price changes, …the reciprocal of which shall vary in the same sense as the economic welfare of the group of persons whose fortunes we are at the time considering.”

A. C. Pigou (1912)
**Weak Ordinal COLI:**

Minimum change in expenditure needed to remain on the same indifference curve in two different time periods or locations

**Method:**

Assuming a constant utility function, use different market baskets (e.g., Laspeyres, Paasche) or combinations of indexes to bound the true index. **Changing utility functions render the calculation meaningless.**

“[C]omparisons of economic welfare must proceed under the hypothesis of constant wants….If this assumption cannot be granted, the question whether [an individual] is better off in one situation or in the other loses all economic meaning.”

Sir John Hicks (1940)
**Strong Ordinal COLI:**

Minimum change in expenditure needed for a consumer with fixed preferences to remain on the same indifference curve when facing two different sets of prices.

**Method:**

Prices are drawn from two different time periods, but there is no comparison of utility or welfare between the two periods. **Changing utility functions present no fundamental obstacles.** Instead, they raise a puzzle (addressed by superlative indexes, Diewert 1976) about which utility functions to use in a calculation or how to combine calculations based on different utility functions.

“Given an indifference map, we compare two hypothetical situations, A and B. We ask how much income the consumer in B would require to make him just indifferent between facing B’s prices and facing A’s prices with a stated income.’ Note that the question of whether the consumer has the same utility in A as in B never arises.”

Fisher & Shell (1972)
<table>
<thead>
<tr>
<th>Cardinal COLI</th>
<th>(Hybrid)</th>
<th>Weak Ordinal COLI</th>
<th>(Hybrid)</th>
<th>Strong Ordinal COLI</th>
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<tbody>
<tr>
<td>Sidgwick (1887)</td>
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<tr>
<td>Edgeworth (1887-89)</td>
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<td>Pigou (1912)</td>
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<td>Bowley (1920, 1928)</td>
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<td>Gini (1924)</td>
<td>Konüs (1924)</td>
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<td>Haberler (1927)</td>
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<td>Keynes (1930)</td>
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<td>Bortkiewicz (1932)</td>
<td>Allen (1933)</td>
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<td>Staehle (1934)</td>
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<td>Frisch (1936)</td>
<td>Hicks (1940)</td>
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<td>Samuelson (1947)</td>
<td>Allen (1949)</td>
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<td>Frisch (1954)</td>
<td>Konüs (1968)</td>
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<td></td>
<td>Fisher &amp; Shell (1972)</td>
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<td>Samuelson &amp; Swamy (1974)</td>
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<td>Gillingham (1974)</td>
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<td>Allen (1975)</td>
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<td>Diewert (1983)</td>
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<td>Pollak (1989)</td>
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Theoretical Naïveté: The 1961 NBER Report

In 1961, an NBER committee, led by George Stigler, famously recommended making the U.S. CPI a constant-utility index.

Yet the committee never discussed what it meant by “welfare” (ordinal or cardinal). Nor did it mention the need to assume constant consumer tastes, although that was a standard stipulation in all contemporary literature.

Most government economists rejected the committee’s recommendations precisely because of these failures.

“There are some very important restriction upon the circumstances under which economic theory permits such a measurement of a [constant utility COLI]…[I]f environment and tastes [have] changed, the theory would not permit the computation of a cost of living change. (Unfortunately, the persons responsible for calculating actual price index numbers cannot bypass such situations).”

Arthur M. Ross, Commissioner, US BLS (1966)
Theoretical Ambiguity: 
The 1996 Boskin Report

In 1996, an advisory committee appointed by the US Senate and led by Michael Boskin again recommended basing the US CPI on a constant-utility index.

To support its recommendation, the committee cited literature that adopted the strong ordinal approach.

Yet although some committee members described the pitfalls of adjusting income payments with such an index, the main report did not explain its theoretical framework clearly. In fact, it mistakenly implied that a strong ordinal index was designed to maintain constant welfare over time:

“[A COLI tells us] how much would we need to increase (or decrease) initial (period 1) expenditure in order to make the consumer as well off as in the subsequent period (period 2).”

Boskin, et. al. (1996)
Potential Misinterpretations of a Strong Ordinal COLI

1. As is well known, the typical COLI does not encompass changes beyond the consumer market (e.g., environmental pollution)

2. However, because the strong ordinal COLI avoids inter-temporal comparisons of economic welfare, it also does not account for how changes in productivity may affect utility functions.

3. In general, one would expect that a strong ordinal COLI will understate changes in expenditures needed to maintain constant (cardinal) welfare in a society with rising productivity.

   “The total well-being we derive from goods depends not only on the positive satisfaction experienced in use or consumption,’ but also ‘on the social satisfactions that flow to us in consequence, the latter largely determined by the relation of our consumption to that of our neighbours.’ …[Therefore,] in a progressive society…[the ongoing increase in productivity] tends to depreciate goods with respect to utility.” Francis Edgeworth (1894)
Consequences of Change in COLI Theory

<table>
<thead>
<tr>
<th>Year</th>
<th>Item</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>2003</td>
<td>Mid-range, 2003 desktop</td>
<td>$800</td>
</tr>
<tr>
<td>2013</td>
<td>Mid-range, 2013 desktop (much faster CPU, more memory, greater storage, better graphics, etc.)</td>
<td>$800</td>
</tr>
</tbody>
</table>

What has happened to the cost of living?

- (Strong Ordinal COLI): Decrease in real price for computing; COLI shows quite substantial fall
- (Cardinal COLI): Stable or slight decrease in COLI
Consequences for COLI Applications

1. The strong ordinal COLI fits with a longstanding tradition in empirical macroeconomics that deliberately omits changes in utility functions. For example:
   - Deflation of consumer expenditures for national accounts
   - Monitoring price controls

2. However, because the strong ordinal COLI avoids inter-temporal comparisons of economic welfare, it is not a suitable tool for adjusting payments to maintain a constant standard of living. Such unsuitable applications might include:
   - Cost-of-living adjustments to pensions or welfare payments
   - Adjustments to tax brackets
   - Inflation-protected bonds
Replacing the Strong Ordinal COLI?

1. Despite the weaknesses of the strong ordinal COLI, there is no straightforward, non-normative substitute.

2. There is no obvious way to implement a cardinal COLI, and indeed it is unclear whether such an index (even if calculable) would be suitable for many applications.

3. For some applications (e.g., adjusting welfare payments), the most practical alternative might be a mixture of methods – e.g., a strong ordinal COLI for some expenditure categories and periodically updated “standard budgets” for others.

4. Theoretical support for such an approach may be based on the concept of “capabilities” that Amartya Sen has proposed substituting for “utility” when assessing economic development.

5. Obviously, any such statistics would entail policy judgments.