Hedonic Price-Rent Ratios, User Cost, and Departures from Equilibrium in the Housing Market

Robert J. Hill\textsuperscript{a} and Iqbal A. Syed\textsuperscript{b,}\textsuperscript{*}

\textsuperscript{a}Department of Economics, University of Graz, Universitätsstrasse 15/F4, 8010 Graz, Austria: robert.hill@uni-graz.at
\textsuperscript{b}School of Economics, University of New South Wales, Sydney 2052, Australia: i.syed@unsw.edu.au

March 01, 2013

Abstract: Disequilibrium in the housing market can be detected by comparing the actual price-rent ratio with its equilibrium counterpart obtained from the user-cost condition. Empirical implementation of this idea, however, is problematic because of quality differences between sold and rented dwellings. We develop a hedonic method that resolves this problem even in the presence of omitted variables. Applying this method to a data set consisting of 730,000 individual price and rent transactions we find that quality adjusting significantly reduces the actual price-rent ratio. We use these quality adjusted price-rent ratios in the user cost condition to check for departures from equilibrium.

Keywords: Housing market; Hedonic model; Price-rent ratio; Rental yield; Quality adjustment; User cost; Capital gains

JEL Classification Codes: C43; E01; E31; R31

* Corresponding author.

We thank Australian Property Monitors and the NSW Department of Housing for providing the data and the Australian Research Council for providing financial assistance for the research (DP0667209 and LP0884095). We also thank Erwin Diewert, Kevin Fox, Tran Van Hoa, Glenn Otto, and Nigel Stapledon for their helpful comments.
Recent events have shown how the housing market can impact on the rest of the economy, as a bust in the US housing market precipitated a global financial crisis. As housing markets are particularly prone to booms and busts, it is particularly important that policy makers and other market participants can detect departures from equilibrium before they become too extreme.

One way of detecting such departures is to compare the user cost of owner-occupying with the cost of renting. In equilibrium, households should be indifferent between these alternatives. When the user cost of owner-occupiers is higher (lower) than the cost of renting, then the price-rent ratio is too high (low). Departures from equilibrium therefore can be detected by comparing actual price-rent ratios with the price-rent ratio derived from the user-cost equilibrium condition.

The equilibrium condition assumes that a household is choosing between owner-occupying and renting dwellings of equal quality. In other words, if there is a quality difference between the sold and rented dwellings this invalidates the user-cost equilibrium condition. One way to avoid quality mismatches in the actual price-rent ratio is to match prices and rents at the level of individual dwellings. In general this is not possible since dwellings sell and rent only at irregular intervals, which typically do not coincide. Household surveys also cannot get information on the actual price and rent of the same dwelling, since a household is either an owner-occupier or a renter.

Quality differences therefore are likely to exist in actual price-rent ratios, especially those based on medians. The existing literature suggests that owner-occupied dwellings are on average of better quality than rented dwellings. For example, according to the American Housing Survey (2001), 82 percent of owner-occupied dwellings are detached single-family homes, while the corresponding figure for rental dwellings is only 23 percent (see also Gallin 2008 and Heston and Nakamura 2009). Also, Shilling, Sirmans and Dombrow (1991) show that owner-occupied dwellings are better maintained than rented dwellings. An implication of this is that applications of the user-cost equilibrium condition based on median prices and median rents (see for example Hatzvi and Otto 2008) will be biased towards finding that the price-rent ratio is above its equilibrium level.

Many applications of the user-cost equilibrium condition compare price and rent indexes rather than median prices and rents. Notable examples include Leamer (2002), Himmelberg, Mayer and Sinai (2005), Girouard, Kennedy, Noord and André (2006), Gallin (2008),
Verbrugge (2008), Campbell, Davis, Gallin and Martin (2009), Duca, Muellbauer and Murphy (2011), and Hiebert and Sydow (2011). As noted by Smith and Smith (2006), such an approach is also problematic.

[T]he dwellings included in price indexes do not match the dwellings in rent indexes, so that the resulting comparison is of apples to oranges. The ratio of a home sale price index to a rent index can rise because the prices of homes in desirable neighborhoods increased more than did the rents of apartment buildings in less desirable neighborhoods. Or perhaps the quality of the average home in the price index has increased relative to the quality of the average property in the rent index. In any case, gauging fundamental value requires actual rent and sale price data, not indexes with arbitrary scales. (p. 7)

Even when the price and rent indexes are themselves quality adjusted, the derived price-rent ratios may not be (see section 3.4). Also the use of price and rent indexes only allows comparisons between the change in the price-rent ratio and the change in its corresponding equilibrium level. At any point in time, therefore, we cannot answer the most fundamental question which is whether the price-rent ratio is above or below its equilibrium level or whether it is moving towards or away from equilibrium.

This apples and oranges problem can be seen clearly in some applications that, due to the lack of more suitable data, combine US price and rent indexes from different sources. Leamer (2002) constructs price-rent ratios by dividing a median house price index by the rent of shelter index from the consumer price index (CPI) produced by the Bureau of Labor Statistics. Himmelberg et al. (2005) divide a repeat-sales price index calculated for single-family houses obtained from the Office of Federal Housing Enterprise Oversight (OFHEO) – now part of the Federal Housing Finance Agency (FHFA) – by an index of annual average rents of two-bedroom apartments obtained from REIS (a real estate consulting firm). Gallin (2008) and Campbell et al. (2009) use the same FHFA repeat-sales price index as Himmelberg et al., and the tenant rent index (part of the rent of shelter index) from the CPI. Duca et al. (2011) divide the FHFA repeat-sales index by the rental fixed dwelling index from the personal consumption expenditure (PCE) price index produced by the Bureau of Economic Analysis.

In this paper we develop a hedonic approach which provides estimates of price-rent
ratios matched at the level of individual dwellings. Our hedonic method adjusts for the quality difference derived from included, missing and omitted characteristics (where a missing characteristic is missing for a particular dwelling but available for other dwellings, while an omitted characteristic is missing for all dwellings in our data set). The correction for omitted characteristics is provided by a subsample of dwellings that both sell and rent during our sample period.

Applying our method to about 730,000 price and rent observations for Sydney, Australia over the period 2001 to 2009 we find that included, missing and omitted characteristics on average generate quality adjustments of 8.7, 3.5 and 6.2 percent respectively. Taken together this yields a total quality difference of 18.6 percent between sold and rented dwellings. The difference for owner-occupied versus rented dwellings is even larger (i.e., 26 percent), since some sold dwellings are subsequently rented. Also, we find that the magnitude of the quality bias is not fixed over the sample period. While reasonably stable from 2001 to 2006 it falls significantly from 2007.

One problem that arises when calculating the equilibrium price-rent ratio is that the expected capital gain on housing - a crucial input into the user-cost equilibrium condition - is not directly observed. We consider two ways of imputing it. First, it can be imputed from the past performance of the housing market. We find that the time horizon over which past performance is measured is crucial. When the time horizon is too short the equilibrium price-rent ratio is prone to become volatile and to rise in booms and fall in busts, both of which effects are liable to undermine the method’s ability to detect departures from equilibrium. We therefore recommend a long time horizon of say 30 years. Using this approach we find that the price-rent ratio in Sydney was above its equilibrium level from 2001 to 2008, although not in 2009. In the absence of quality adjustment, the departure from equilibrium seems even larger than it actually was.

Alternatively, the expected capital gain implied by market equilibrium can be imputed directly from the user-cost equilibrium condition. If this is deemed implausibly high or low, then the assumption of equilibrium can be rejected. On average over our sample period, the imputed expected real capital gain required for the Sydney housing market to be in equilibrium is 4.0 percent, rising to 4.6 percent in the absence of quality adjustment. Compared with other cities 4.0 percent seems too high, thus again leading to the conclusion that the price-rent ratio was above its equilibrium level.
We also use our individual dwelling data to explore how the price-rent ratio varies over the housing distribution. We find that it is systematically higher at the top end. We argue that the same is probably true as well for the equilibrium price-rent ratio. An important implication of this is that, when assessing whether or not the housing market is in equilibrium, it is not enough to simply focus on the median.

More generally, our methodology and results also have applications that extend beyond the main issues addressed here. For example, failure to account for the quality difference between owner-occupied and rented dwellings and cross-section variation in the price-rent ratio may result in the flow of housing services (and hence GDP) being mismeasured.

The remainder of this paper consists of six sections. Section 1 develops our hedonic approach for computing price-rent ratios at the level of individual dwellings. Section 2 describes our data set, and then explains our methods for correcting for missing characteristics and omitted variables. Our estimates of quality bias in actual price-rent ratios are presented in section 3. Section 4 explains the user-cost equilibrium condition, and then uses it to check for departures from equilibrium. Some implications of our findings for the measurement of GDP are considered in section 5. Finally, our conclusions are discussed in section 6.

1 A Hedonic Approach to Constructing Quality-Adjusted Price-Rent Ratios

1.1 The hedonic imputation method

The hedonic method dates back at least to Waugh (1928) and Court (1939). It was, however, only after Griliches (1961) that hedonic methods started to receive serious attention (see Schultze and Mackie 2002 and Triplett 2006). The conceptual basis of the approach was laid down by Lancaster (1966) and Rosen (1974). The hedonic model is a reduced form equation which regresses the price of a product on a vector of characteristics (whose prices are not independently observed).

The hedonic approach can be implemented in different ways (see Triplett 2006 and Hill 2013 for surveys of the literature). However, in our context the most appropriate method is the hedonic imputation method where a separate hedonic model is estimated for each
comparison period typically using a semilog functional form.\(^1\)

\[ y_t = X_t \beta_t + u_t, \]  

where \( y_t \) is an \( H_t \times 1 \) vector with elements \( y_h = \ln p_h \) (where \( H_t \) denotes the number of dwellings sold in period \( t \)), \( X_t \) is an \( H_t \times C \) matrix of characteristics (some of which may be dummy variables), \( \beta_t \) is a \( C \times 1 \) vector of characteristic shadow prices, and \( u_t \) is an \( H_t \times 1 \) vector of random errors. Examples of characteristics include the number of bedrooms, number of bathrooms, land area, and postcode.

Once the hedonic model has been estimated separately for each period, the prices of dwellings sold in one period can be imputed from the hedonic model of another period. For example, let \( \hat{\tilde{p}}_{th}(x_{sh}) \) denote the estimated price in period \( t \) of a dwelling \( h \) sold in period \( s \). This price is imputed by substituting the characteristics of dwelling \( h \) into the estimated hedonic model of period \( t \) as follows:

\[
\hat{\tilde{p}}_{th}(x_{sh}) = \exp\left( \sum_{c=1}^{C} \hat{\beta}_{ct} x_{csh} \right),
\]

where \( c = 1, \ldots, C \) indexes the set of characteristics included in the hedonic model. A Laspeyres-type hedonic index that compares periods \( s \) and \( t \) using the dwellings sold in period \( s \) can now be constructed in one of two ways:

\[
L1 : \quad P_{st}^{L1} = \sum_{h=1}^{H_s} w_{sh} \left[ \hat{\tilde{p}}_{th}(x_{sh})/\bar{p}_{sh} \right] = \sum_{h=1}^{H_s} \hat{\tilde{p}}_{th}(x_{sh})/\sum_{h=1}^{H_s} \bar{p}_{sh}
\]

\[
L2 : \quad P_{st}^{L2} = \sum_{h=1}^{H_s} \hat{w}_{sh} \left[ \hat{\tilde{p}}_{th}(x_{sh})/\hat{\tilde{p}}_{sh}(x_{sh}) \right] = \sum_{h=1}^{H_s} \hat{\tilde{p}}_{th}(x_{sh})/\sum_{h=1}^{H_s} \hat{\tilde{p}}_{sh}(x_{sh}),
\]

where \( w_{sh} \) and \( \hat{w}_{sh} \) denote actual and imputed expenditure shares calculated as follows:

\[
w_{sh} = p_{sh}/\sum_{m=1}^{H_s} p_{sm}, \quad \hat{w}_{sh} = \hat{\tilde{p}}_{sh}(x_{sh})/\sum_{m=1}^{H_s} \hat{\tilde{p}}_{sm}(x_{sm}).
\]

In an analogous manner corresponding Paasche-type hedonic indexes that compare periods \( s \) and \( t \) using the dwellings sold in period \( t \) can be constructed:

\[
P_{st}^{P1} = \left\{ \sum_{h=1}^{H_t} w_{th} [p_{th}/\bar{p}_{sh}(x_{th})]^{-1} \right\}^{-1} = \sum_{h=1}^{H_t} p_{th}/\sum_{h=1}^{H_t} \bar{\tilde{p}}_{sh}(x_{th})
\]

\[
P_{st}^{P2} = \left\{ \sum_{h=1}^{H_t} \hat{w}_{th} [\hat{\tilde{p}}_{th}(x_{th})/\hat{\tilde{p}}_{sh}(x_{sh})]^{-1} \right\}^{-1} = \sum_{h=1}^{H_t} \hat{\tilde{p}}_{th}(x_{th})/\sum_{h=1}^{H_t} \hat{\tilde{p}}_{sh}(x_{th}).
\]

A Fisher-type hedonic index, that treats periods \( s \) and \( t \) symmetrically, is obtained by taking

\(^1\)Alternative functional forms, such as linear or Box-Cox transformations, are sometimes also considered. See Diewert (2003) and Malpezzi (2003) for a discussion of some of the advantages of semilog in a hedonic context and Diewert, Heravi and Silver (2009) for advantages of the hedonic imputation method.
the geometric mean of Laspeyres and Paasche:

$$F1 : P_{st}^{F1} = \sqrt{P_{st}^{L1} \times P_{st}^{L1}} = \sqrt{\frac{\sum_{s=1}^{H_s} \hat{p}_{th}(x_{sh})}{\sum_{s=1}^{H_s} \hat{p}_{sh}(x_{sh})} \times \frac{\sum_{t=1}^{H_t} p_{th}(x_{th})}{\sum_{t=1}^{H_t} \hat{p}_{sh}(x_{th})}}; \quad (4)$$

$$F2 : P_{st}^{F2} = \sqrt{P_{st}^{L2} \times P_{st}^{L2}} = \sqrt{\frac{\sum_{s=1}^{H_s} \hat{p}_{th}(x_{sh})}{\sum_{s=1}^{H_s} \hat{p}_{sh}(x_{sh})} \times \frac{\sum_{t=1}^{H_t} p_{th}(x_{th})}{\sum_{t=1}^{H_t} \hat{p}_{sh}(x_{th})}}. \quad (5)$$

In the hedonic literature L1, P1 and F1 are referred to as single imputation price indexes, and L2, P2 and F2 as double imputation price indexes (see Triplett 2006 and Hill and Melser 2008). No clear consensus has emerged in the literature as to which approach is better. Single imputation uses less imputations and therefore is preferred by statistical agencies (see de Haan 2004). Double imputation may reduce omitted variables bias (see Hill and Melser 2008). We find that for our data set both the F1 and F2 price indexes and the F1 and F2 rent indexes are almost indistinguishable.

### 1.2 Hedonic price-rent ratios for individual dwellings

Here we apply the logic of the hedonic imputation method in a new context. Our objective is to compute a matched price-rent ratio for each individual dwelling. We achieve this by first estimating separate price and rent hedonic models. A price for each rented dwelling can then be imputed from the hedonic price model, and a rent for each sold dwelling imputed from the hedonic rent model. In this way a price-rent ratio can be calculated for each rented dwelling and each sold dwelling. A feature of this approach is that the hedonic price and rent models need to be defined on the same set of characteristics.

Papers that have implemented some of these steps include Arévalo and Ruiz-Castillo (2006), Kurz and Hoffmann (2009), Crone, Nakamura and Voith (2009) and Davis, Lehnert and Martin (2008). Among these papers, only Davis et al. consider estimation of price-rent ratios. Using US Census survey data, they impute rents for individual dwellings from a hedonic model. These rents are then matched with price estimates for these same dwellings obtained directly from the survey. The rent-price ratio is then averaged and interpolated from one Census benchmark to the next.

The methodological scope of our paper is broader than Davis et al. in that (as noted above) it estimates both price and rent hedonic models and then uses them to impute a rent for each dwelling sold and a price for each dwelling rented. Price-rent ratios at the level of individual dwellings are hence calculated using a double-imputation approach. More
importantly, we develop extensions of our basic method to account for missing characteristics (i.e., characteristics that are missing for only some dwellings in our data set) and omitted variables (i.e., characteristics that are missing for all dwellings in our data set). These issues are addressed in sections 2.2 and 2.3. Likewise, our empirical focus is broader in that we consider both the cross-section of price-rent ratios and the evolution of the average over time, and then insert the average into the user cost formula to detect departures from equilibrium.

Our starting point is the hedonic price equation, which is assumed to take the following form:

\[ y_{Pt} = X_{Pt}\beta_{Pt} + u_{Pt}, \]  

(6)

where \( y_{Pt} \) is the vector of log prices of the dwellings sold in period \( t \), and \( X_{Pt} \) is the corresponding matrix of sold dwelling characteristics and \( u_{Pt} \) is the random error term with zero mean and a constant variance. Similarly, the hedonic rent equation is as follows:

\[ y_{Rt} = X_{Rt}\beta_{Rt} + u_{Rt}, \]  

(7)

where \( y_{Rt} \) is the vector of log rents of the dwellings rented in period \( t \), and \( X_{Rt} \) is the corresponding matrix of rented dwelling characteristics. A rent for each dwelling \( h \) sold in period \( t \) is imputed from (7) as follows:

\[ \hat{\ln r}_{th} = \sum_{c=1}^{C} \hat{\beta}_{Rtc} x_{Pthc}, \]  

(8)

where \( c = 1, \ldots, C \) indexes the list of characteristics over which the price and rent hedonic models are defined. Similarly, a price for each dwelling \( j \) rented in period \( t \) is imputed from (6) as follows:

\[ \ln p_{tj} = \sum_{c=1}^{C} \hat{\beta}_{Ptc} x_{Rtjc}. \]  

(9)

We can also use the hedonic rent equation to impute a rent for a dwelling \( j \) actually rented in period \( t \):

\[ \ln \hat{r}_{tj} = \sum_{c=1}^{C} \hat{\beta}_{Rtc} x_{Rtjc}, \]  

(10)

and the hedonic price equation to impute a price for a dwelling \( h \) actually sold in period \( t \):

\[ \ln \hat{p}_{th} = \sum_{c=1}^{C} \hat{\beta}_{Ptc} x_{Pthc}. \]  

(11)

\(^2\)Part of the omitted variables problem may take the form of quality differences in the included characteristics (e.g., number of bedrooms and number of bathrooms) across the price and rent data sets.

\(^3\)Spatial dependence in our model is captured through the inclusion of postcode dummies. With data on individual dwelling longitudes and latitudes, the spatial dependence could be modeled more rigorously using a spatial autoregressive model with autoregressive errors (see for example Badinger and Egger 2011).
Exponentiating, it follows that:\footnote{Strictly speaking, \( \hat{r} \) and \( \hat{p} \) are biased estimates of \( r \) and \( p \) since by exponentiating we are taking a nonlinear transformation of a random variable. For example, \( \hat{p}_{ij}(x_{Pth}) = \exp \left( \sum_{c=1}^{C} \hat{\beta}_{Ptc} x_{Pthc} \right) \times \exp(u_{Pth}), \) where \( E \exp(u_{Pth}) \neq 0. \) Possible corrections have been proposed by Goldberger (1968), Kennedy (1981) and Giles (1982). From our experience, however, these corrections are small enough that they can be ignored.}

\[
\begin{align*}
\hat{r}_{th}(x_{Pth}) &= \exp \left( \sum_{c=1}^{C} \hat{\beta}_{Rtc} x_{Pthc} \right), \\
\hat{p}_{ij}(x_{Rtj}) &= \exp \left( \sum_{c=1}^{C} \hat{\beta}_{Ptc} x_{Rtjc} \right), \\
\hat{r}_{ij}(x_{Rtj}) &= \exp \left( \sum_{c=1}^{C} \hat{\beta}_{Rtc} x_{Rtjc} \right), \\
\hat{p}_{ij}(x_{Pth}) &= \exp \left( \sum_{c=1}^{C} \hat{\beta}_{Ptc} x_{Pthc} \right).
\end{align*}
\]

The distinction between single and double imputation arises again in the calculation of our hedonic price-rent ratios. A single imputation price-rent ratio \( P/R(sold)_{th}^{SI} \) for a dwelling \( h \) sold in period \( t \) divides the actual price at which dwelling \( h \) is sold by its imputed rent in period \( t \) obtained from (8):

\[
P/R(sold)_{th}^{SI} = \frac{P_{th}}{\hat{r}_{th}(x_{Pth})} = \frac{P_{th}}{\exp \left( \sum_{c=1}^{C} \hat{\beta}_{Rtc} x_{Pthc} \right)}.
\]

A corresponding double imputation price-rent ratio \( P/R(sold)_{th}^{DI} \) divides the imputed price for dwelling \( h \) obtained from (11) by its imputed rent obtained from (8):

\[
P/R(sold)_{th}^{DI} = \frac{\hat{p}_{th}(x_{Pth})}{\hat{r}_{th}(x_{Pth})} = \frac{\exp \left( \sum_{c=1}^{C} \hat{\beta}_{Ptc} x_{Rthc} \right)}{\exp \left( \sum_{c=1}^{C} \hat{\beta}_{Rtc} x_{Pthc} \right)}.
\]

(12)

We can likewise generate two alternative matched price-rent ratios for each dwelling \( j \) rented in period \( t \). A single imputation price-rent ratio \( P/R(rented)_{ij}^{SI} \) divides the imputed price for dwelling \( j \) obtained from (9) by its actual rent:

\[
P/R(rented)_{ij}^{SI} = \frac{\hat{p}_{ij}(x_{Pth})}{r_{ij}} = \frac{\exp \left( \sum_{c=1}^{C} \hat{\beta}_{Ptc} x_{Rtjc} \right)}{r_{ij}}.
\]

Finally, a double imputation price-rent ratio \( P/R(rented)_{ij}^{DI} \) divides the imputed price for dwelling \( j \) obtained from (9) by its imputed rent obtained from (10):

\[
P/R(rented)_{ij}^{DI} = \frac{\hat{p}_{ij}(x_{Rtj})}{\hat{r}_{ij}(x_{Rtj})} = \frac{\exp \left( \sum_{c=1}^{C} \hat{\beta}_{Ptc} x_{Rtjc} \right)}{\exp \left( \sum_{c=1}^{C} \hat{\beta}_{Rtc} x_{Rtjc} \right)}.
\]

(13)

Empirically, we find that on average our double imputation price-rent ratios are 3.4 percent lower than their corresponding single-imputation counterparts (see column b of Table 10). The choice between single and double imputation methods does not affect the general
thrust of our results in section 3. The subsequent analysis focuses on the double imputation price-rent ratios.\footnote{A robustness check is also conducted where the hedonic models in (6) and (7) are specified on price and rent levels instead of the log of prices and rents. In this case, (12) is replaced by $P/R(sold)^{DI}_j = \sum_{c=1}^{C} \hat{\beta}_{Ptc}x_{Ptchc}/\sum_{c=1}^{C} \hat{\beta}_{Ptchc}$ and (13) by $P/R(rented)^{DI}_j = \sum_{c=1}^{C} \hat{\beta}_{Rtc}x_{Rtjc}/\sum_{c=1}^{C} \hat{\beta}_{Rtjc}$. The price-rent ratios obtained from the level models are on average about the same as those obtained from the log models (see column c of Table 10).}

### 1.3 Median matched price-rent ratios

Let $Med[P/R(sold)^{DI}]$ denote the median price-rent ratio derived from the double-imputation price-rent distribution defined on the dwellings actually sold, while $Med[P/R(rented)^{DI}]$ denotes the corresponding median price-rent ratio defined on the dwellings actually rented. An overall median is obtained by averaging these two population specific medians as follows:

$$Med[P/R^{DI}] = \sqrt{Med[P/R(sold)^{DI}] \times Med[P/R(rented)^{DI}].}$$

(14)

An alternative approach is to first pool the price-rent distributions defined on sold and rented dwellings and then calculate the median.

$$Med[P/R_{pooled}^{DI}] = Med[P/R(sold)^{DI}, P/R(rented)^{DI}]$$

Intuitively, we prefer the former approach (i.e. averaging rather than pooling) in (14) since it gives equal weight to the price and rent data sets. Empirically we find that the averaged and pooled medians are very close (see column d of Table 10).

### 2 Data Sets and Empirical Strategy

#### 2.1 The hedonic price and rental data sets

The data sets used in this paper are for Australia’s largest city, Sydney, over the period 2001 to 2009. These are assembled from three sources. The data pertain to separate houses, where each house is built on its own piece of land. The data set on actual transaction prices is obtained from Australian Property Monitors (APM) and consists of a total of 395,110 observations over the 2001 to 2009 period.\footnote{APM provides real estate related research service and data for the Australian market. See http://apm.com.au in order obtain access to their data sets.} The characteristics included in the data set...
are the transaction price, exact date of sale, land area, number of bedrooms, number of bathrooms, exact address and a postcode identifier. The rental data set is obtained by combining rental data from the New South Wales (NSW) Department of Housing (of which we have 331,877 observations) with data from APM (of which we have 89,495 observations that are not also in the NSW Housing data set). In total, therefore, we have 421,372 rental observations.\footnote{While the recorded rents in the NSW Housing data refer to new rental contracts, the rents in the APM data refer to rents as advertised in the media. However, we find that there is virtually no difference between the actual and advertised rents when we test their mean difference on the houses which appeared in both data sets in a given quarter.}

A problem with the data sets is that there are many observations for which one or more characteristics are missing, even after filling in some missing values through the matching of addresses across the three data sets. In particular, all the characteristics are available for 61.67 percent of the price data and for 45.60 percent of the rental data (see Table 1). For the remainder, at least one of the three characteristics of land area, number of bedrooms and number of bathrooms is missing. We explain in section 2.2 how we deal with this problem.

**Insert Table 1 Here**

With regard to the nature of the missing data, there are reasons to believe that values are randomly missing in the sense that a particular missing value is not related to the value itself. The original sources are government agencies (except for the APM rental data). The NSW Valuer-General’s Office updates record of holding of properties and assesses the value of land which is used to determine property tax. The NSW Rental Bond Authority stores bonds (deposits provided by the tenants) and acts as an intermediary in the case of disputes between landlords and tenants. In each case the physical characteristics information are not so important. The key information for these agencies are addresses, prices, rents, bond amount, contract date, name of owners and renters). It does not benefit any party to withhold characteristic information (such as the number of bedrooms). Characteristic information can go missing both at the submission and data entry stage. APM, however, has supplemented the data with characteristic information obtained from other sources (such as newspapers, online housing databases and real estate offices). Similarly, APM’s rental data are obtained from advertisements posted on online websites and at real-estate offices. While this process of supplementing the existing databases could in theory cause the missing characteristics to
no longer be random, there is no particular reason to expect such an occurrence.\textsuperscript{8}

The data sets are expected to provide a comprehensive picture of the purchase and rental markets in Sydney. It is mandatory for the parties to inform the State Valuer-General in the case of any change in the ownership of a property. The Rental Bond Authority obtains the information on new rental contract when the renter deposits the amount of bond with the agency. The authority does not charge any party for their service. While it is not mandatory, most new contracts are recorded with the Rental Bond Authority. Many of the contracts not lodged with the Authority are captured in the APM rental data.

Before proceeding with the estimation of our hedonic models, we removed some extreme observations (most of which are data-entry errors). The houses whose recorded prices, rents and land areas are located in the extreme 1.0 percent in both tails of the distributions are deleted. In addition, houses with the number of bedrooms greater than 6 and the number of bathrooms greater than 5 are also deleted (these correspond to the 99.68 and 99.95 percentiles in the price data and 99.99 and 99.99 in the rent data). We had to undertake some further deletions in order to implement our hedonic approach since it requires that both price and rent models are specified on the same set of characteristics. For example, if the hedonic price model includes dwellings in a particular postcode, then the rental model must include dwellings rented in the same postcode.\textsuperscript{9} In total, the deletion of extreme observations and the deletions due to the matching requirement led to the exclusion of 10.6 percent of observations from the total number of price and rental observations. This leaves us with 371,604 observations in the price data and 358,381 observations in the rental data (see Table 1 for detailed descriptions of the data sets).

Our expectation is that owner-occupied (and hence sold) dwellings on average are of

\textsuperscript{8}The percentage of missing characteristics has decreased (i.e. the quality of the data has improved) over the years in the sample. However, this pattern should not affect our results because we conducted our regression analysis separately for each year of the data.

\textsuperscript{9}If we had used a larger geographical area, such as ‘local government area’ instead of postcodes, we would have needed to delete fewer observations. However, using a larger area worsens the quality of the matches when adjusting for quality difference between sold and rented dwellings. We could also have tried matching at a lower level of aggregation, such as suburbs, where the suburbs typically cover smaller geographical regions than postcodes. The choice of postcode as the location-specific hedonic characteristic, however, is a natural one, partly because the presence of postcode is universal in addresses and also because postcodes are not prone to mismatches due to name abbreviations (which happens in the case of suburbs). With further improvement in data quality, matching at suburb level may in future become more feasible.
better quality than rented dwellings.\footnote{After a house is sold it can be either occupied by the new owner or rented. ABS (2010) reports that the home-ownership rate in Australia remained stable at around 70 percent over the period 1971-2006. This indicates that 70 percent of the houses sold in each year can be expected to be occupied by the new owner. The home-ownership rate in Australia is similar to that of other countries including Canada, New Zealand, the European Union (EU) and the US (see AFTF 2007, Eurostat 2011, and Sinai and Souleles 2005).} This hypothesis is supported by the figures in Tables 1 and 2. From Table 1 it can be seen that the mean number of bedrooms and bathrooms and mean land area are all higher for sold dwellings than for rental dwellings. Table 2 compares the bedroom, bathroom, land area and locational distributions of the price and rental data. Of particular interest in Table 2 are the locational distributions. These were constructed by ranking the postcodes from cheapest to most expensive in terms of their median prices and median rents, and then allocating the postcodes to decile groups (i.e., the first decile is the cheapest and the tenth is the most expensive). From Table 2, it is clear that the rented dwellings are concentrated relatively more in the cheaper postcodes.

\textbf{Insert Table 2 Here}

While these results support the hypothesis that sold dwellings are of better quality than rented dwellings, the quality differences are not that large. When imputing prices for rented dwellings from the price equation and rents for sold dwellings from the rent equation, the mean values of the characteristics corresponding to the predicted dwellings are quite close to the mean values of the characteristics that enter in the corresponding hedonic equations (see Table 1). These factors combined with our large sample size indicate that our approach of imputing prices for rented dwellings and rents for sold dwellings is viable.

\subsection*{2.2 Imputing prices and rents for dwellings with missing characteristics}

Dwellings with missing characteristics are a common problem in housing data sets.\footnote{For example, Crone, Nakamura and Voith (2009) mention that they experience this problem with the American Housing Survey (AHS) data set that they use.} Instead of deleting these observations, we develop an alternative way of dealing with this issue that may be also applicable in other contexts (such as when estimating price and rent indexes and equivalent rent for owner-occupied houses) and to other data sets (such as electronics data used to construct quality-adjusted price indexes).
Our solution entails estimating a number of different versions of our basic hedonic price and rent equations. This allows the price and rent for each dwelling to be imputed from a hedonic equation that is tailored to its particular mix of available characteristics.

More specifically, focusing on the the case of the hedonic price equation, we estimate the following eight hedonic models (HM1,...,HM8) for each year in our data set:

(HM1): \[ \ln \text{price} = f(\text{quarter dummy, land area, squared land area, num bedrooms, num bathrooms, postcode, land area & bedroom inter., land area & bathroom inter.}) \]

(HM2): \[ \ln \text{price} = f(\text{quarter dummy, num bedrooms, num bathrooms, postcode}) \]

(HM3): \[ \ln \text{price} = f(\text{quarter dummy, land area, squared land area, num bathrooms, postcode, land area & bathroom inter.}) \]

(HM4): \[ \ln \text{price} = f(\text{quarter dummy, land area, squared land area, num bedrooms, postcode, land area & bedroom inter.}) \]

(HM5): \[ \ln \text{price} = f(\text{quarter dummy, num bathrooms, postcode}) \]

(HM6): \[ \ln \text{price} = f(\text{quarter dummy, num bedrooms, postcode}) \]

(HM7): \[ \ln \text{price} = f(\text{quarter dummy, land area, postcode}) \]

(HM8): \[ \ln \text{price} = f(\text{quarter dummy, postcode}) \]

Each of these eight models is estimated using all the available dwellings that have at least these characteristics. For example, a dwelling for which land area, number of bedrooms and number of bathrooms are all available is included in all eight regressions. A dwelling that is missing the land area is included only in HM2, HM5, HM6, and HM8. A dwelling that is missing land area and number of bathrooms is included only in HM6 and HM8, etc.

The imputed price for each dwelling that is entered into (12) and (13), however, is only taken from the equation that exactly matches its list of available characteristics. This means that a dwelling for which all characteristics are available will have its price imputed from HM1. A dwelling that is missing only land area will have its price imputed from HM2. A dwelling missing land area and number of bathrooms will have its price imputed from HM6, etc.

The imputed rents are obtained in an analogous manner from 8 versions of the hedonic rent equation. If we had only estimated the HM1 model, then the price-rent ratios of a large number of dwellings could not have been calculated. Estimating multiple versions of our hedonic model allows us to calculate the price-rent ratio of every dwelling in the data sets.
2.3 Correcting for omitted variables bias

Omitted variables are a problem in all our hedonic models, even in HM1. The omitted variables may be physical (e.g., the quality of the structure, its energy efficiency, the general ambience, floor space, sunlight, the availability of parking, and the convenience of the floor plan), or locational (e.g., street noise, air quality and the availability of public transport links). Omitted variables bias may also result from nonequivalence between the bedroom and bathroom characteristics in the price and rent data sets. For example, a bathroom in a sold dwelling may on average be of better quality than a bathroom in a rented dwelling.

These two sources of omitted variables bias should reinforce each other since both the included and omitted characteristics are likely to be of better quality in the price data set than in the rent data set. This implies that our hedonic price-rent ratios will fail to fully adjust for quality differences.

Our first step in correcting for omitted variables bias is to obtain reference quality-adjusted price-rent ratios that are free of bias. This can be done by collecting dwellings that are both sold and rented over our sample period. We use a house price index and rent index to extrapolate forwards and backwards prices and rents on the same dwelling in different quarters. For example, suppose dwelling $h$ sells in period $s$ at the price $p_{sh}$ and is rented in period $t$ at the rate $r_{th}$. An address-matched price-rent ratio for this dwelling in period $s$ can be calculated by extrapolating the rental rate back to period $s$ using a rental index $R_{st}$ as follows:

$$\frac{P}{R}_{AM}^{sh} = \frac{p_{sh} \times R_{st}}{r_{th}}, \quad (15)$$

or by extrapolating the selling price forward to period $t$ using a price index $P_{st}$ as follows:

$$\frac{P}{R}_{th}^{AM} = \frac{p_{sh} \times P_{st}}{r_{th}}. \quad (16)$$

We now pool all the price-rent ratios derived using (15) and (16), and take the median for each period $t$:

$$AMm(AM_{s_t}) = Med_{h=1,...,H_t}[\frac{P}{R}_{th}^{AM}], \quad (17)$$

---

12The impact of some locational characteristics can sometimes be captured by locational dummies, as long as the geographical zones are sufficiently small. This should be the case for the postcode dummies used here (there are about 213 postcodes in Sydney). Many studies (such as Arévalo and Ruiz-Castillo 2006, Davis et al. 2008, Crone et al. 2009, and Kurz and Hoffmann 2009), however, use locational dummies defined on only a few divisions of a large metropolitan area. These zones are probably too big and heterogeneous to effectively capture locational effects.
where $h = 1, \ldots, H_t$ indexes all the address-matched price-rent ratios in period $t$ in our data set. The notation $AM s_t$ in (17) stands for “address-matched sample”, while $AMm$ stands for “address-matched model”. $AMm(AM s_t)$ should by construction be free of omitted variables bias.\footnote{For dwellings with multiple prices and rents in our sample, we select the chronologically closest price and rent observations to construct our address-matched price-rent ratio. Alternatively, we could consider each price-rent pair. For example, 12 address-matched price-rent ratios can be constructed from (15) and (16) for a dwelling that sold three times and rented twice in our data set. Our concern with this approach is that dwellings with multiple prices and rents may exert too much influence on (17). We try both approaches and find that they generate similar price-rent ratios (see column f of Table 10). For dwellings that sell and rent in the same period, we count these price-rent ratios twice. Hence we have exactly two address-matched price-rent ratios for each dwelling that both sold and rented.}

The price and rent indexes in (15) and (16) are calculated using the repeat-rent and repeat-sales index formulas. We use Calhoun’s (1996) method, which corrects for heteroscedasticity by giving greater weight to repeats that are chronologically closer together (see also Hill, Melser and Syed 2009). Here we prefer the repeat rents/sales method over a hedonic method for computing the rent and price indexes since the former should be free of omitted variables bias. As a robustness check though we also estimate address-matched price-rent ratios using the price and rent indexes obtained from the double imputation hedonic (F2) method. The two approaches generate similar price-rent ratios (see column e of Table 10).

With our methodology in place for constructing quality-adjusted price-rent ratios that are free of omitted variables bias, we can now compute bias correction factors for models HM1, . . . , HM8. We consider first the bias of our HM8 model, since it is the only one of our hedonic models that can be calculated over the full data set. We calculate this as follows:

$$
\lambda_{t, HM8} = \frac{HM 8 m (AM s_t)}{AMm(AM s_t)},
$$

(18)

where $HM 8 m (AM s_t)$ denotes the median price-rent ratio obtained from (14) using the hedonic model HM8 applied to the address-matched sample (AMs) in period $t$. More precisely, we estimate the HM8 model over the full data set and then pick out the imputed price-rent ratios for dwellings in the address-matched sample (AMs). The median is then calculated only over the imputed price-rent ratios in the address-matched sample. The median in the denominator of (18) [i.e., $AMm(AM s_t)$] is obtained from (17).

The samples used to calculate the numerator and denominator of (18) are matched in
two senses. First, the full sample is used to impute prices and rents in the HM8 hedonic model and to calculate the price and rent indexes underlying AMm(AMs\textsubscript{t}). Second, both medians HM8m(AMs\textsubscript{t}) and AMm(AMs\textsubscript{t}) are then calculated only over the address-matched samples. Therefore, while HM8m(AMs\textsubscript{t}) and AMm(AMs\textsubscript{t}) separately may suffer from sample selection bias, any such biases should be largely offsetting in (18).

Any systematic deviation of \(\lambda_{t,HM8}\) from 1 can hence be attributed to omitted variables bias in the HM8 median price-rent ratio. In our empirical results we find in every year that \(\lambda_{t,HM8} > 1\), indicating that omitted variables bias is causing the price-rent ratios obtained from the HM8 model to be systematically too high.\(^{14}\)

The omitted variables bias for each of our other models HM\(j\) (where \(j = 1, \ldots, 7\)) relative to HM8 is calculated as follows:

\[
\lambda_{t,HMj|HM8} = \frac{HMjm(HMjs\textsubscript{t})}{HM8m(HMjs\textsubscript{t})}.
\] (19)

That is, we compare the median price-rent ratio obtained from model HM\(j\) estimated over the HM\(j\) sample with the median price-rent ratio obtained from HM8 estimated over the HM\(j\) sample. We use HM8 (i.e., the model with the least characteristics) as our reference hedonic model since it can be estimated on any subsample of the data set.

Given that the median imputed price-rent ratios \(HMjm(HMjs\textsubscript{t})\) and \(HM8m(HMjs\textsubscript{t})\) in (19) are calculated over the same sample of dwellings (i.e., the HM\(j\) sample), any systematic deviation of \(\lambda_{t,HMj|HM8}\) from 1 can be attributed to missing characteristics in the HM\(j\) model. While both \(HMjm(HMjs\textsubscript{t})\) and \(HM8m(HMjs\textsubscript{t})\) will be subject to bias, our expectation is that the bias will be bigger for \(HM8m(HMjs\textsubscript{t})\) than for \(HMjm(HMjs\textsubscript{t})\) (for \(j = 1, \ldots, 7\)). The other models include more characteristics than HM8. Given our hypothesis that sold dwelling perform better than rental dwellings on these characteristics, it follows that \(\lambda_{t,HMj|HM8}\) should be systematically less than 1. Our empirical results confirm this finding.\(^{15}\)

Our estimate of the overall omitted variables bias of HM\(j\) is then given by:

\[
\lambda_{t,HMj} = \lambda_{t,HM8} \times \lambda_{t,HMj|HM8}.
\] (20)

That is, first we calculate the omitted variables bias of HM8 (i.e., \(\lambda_{t,HM8}\)), and then we

\(^{14}\) As a robustness check, we also compute the denominator in equation (18) using hedonic imputation price and rent indexes. The results are quite similar (see column g of Table 10).

\(^{15}\) As a robustness check we also try using HM1 as the reference sample, where \(\lambda_{t,HMj|HM8} = HMjm(HM1s)/HM8m(HM1s)\). Again this has little impact on the results (see column h of Table 10).
calculate the bias of model HMj relative to that of HM8 (i.e., $\lambda_{t,HMj|HM8}$). The overall omitted variables bias of model HMj is then obtained by multiplying $\lambda_{t,HMS}$ by $\lambda_{t,HMj|HM8}$.

Our expectation is that $\lambda_{t,HMj} < \lambda_{t,HMS}$ for $j = 1, \ldots, 7$ since as already noted each of these other models has less omitted variables. Applying the same logic we should also expect that:

$$
\begin{align*}
\lambda_{t,HM1} &< \lambda_{t,HM2} < \lambda_{t,HM5} < \lambda_{t,HMS}; \\
\lambda_{t,HM1} &< \lambda_{t,HM2} < \lambda_{t,HM6} < \lambda_{t,HMS}; \\
\lambda_{t,HM1} &< \lambda_{t,HM3} < \lambda_{t,HM5} < \lambda_{t,HMS}; \\
\lambda_{t,HM1} &< \lambda_{t,HM3} < \lambda_{t,HM7} < \lambda_{t,HMS}; \\
\lambda_{t,HM1} &< \lambda_{t,HM4} < \lambda_{t,HM6} < \lambda_{t,HMS}; \\
\lambda_{t,HM1} &< \lambda_{t,HM4} < \lambda_{t,HM7} < \lambda_{t,HMS}.
\end{align*}
$$

(21)

For example, taking the first of these inequalities, we have that HM2 is obtained by deleting land area from HM1. HM5 is then obtained from HM2 by deleting number of bedrooms. Finally, HM8 is obtained by deleting number of bathrooms.

We therefore adjust the price-rent ratio of a dwelling $h$ sold in period $t$ with the HMj mix of characteristics for omitted variables bias by dividing it by $\lambda_{t,HMj}$ as follows:

$$
P/R(\text{sold})_{th,HMj}^{adj} = \frac{P/R(\text{sold})_{th,HMj}}{\lambda_{t,HMj}} = \frac{P/R(\text{sold})_{th,HMj}}{\lambda_{t,HMj|HM8} \times \lambda_{t,HMS}} = P/R(\text{sold})_{th,HMj} \times \left(\frac{AMm(AMs_t)}{HM8m(AMs_t)}\right) \times \left(\frac{HM8m(HMjs_t)}{HMjm(HMjs_t)}\right).
\tag{22}
$$

Similarly, a dwelling $j$ with the HMj mix of characteristics rented in period $t$ is adjusted for omitted variables bias as follows:

$$
P/R(\text{rented})_{tj,HMj}^{adj} = \frac{P/R(\text{rented})_{tj,HMj}}{\lambda_{t,HMj}} = \frac{P/R(\text{rented})_{tj,HMj}}{\lambda_{t,HMj|HM8} \times \lambda_{t,HMS}} = P/R(\text{rented})_{tj,HMj} \times \left(\frac{AMm(AMs_t)}{HM8m(AMs_t)}\right) \times \left(\frac{HM8m(HMjs_t)}{HMjm(HMjs_t)}\right).
\tag{23}
$$

3 Empirical Results

3.1 The estimated hedonic models

We estimate our eight versions of the price and rent hedonic models, HM1–HM8, separately for each of the 9 years in the data set (altogether 144 regressions are run). Focussing on the
HM1 model first, which is our most general model, Table 3 provides the average results of some key statistics for the 9 yearly regressions, separately for the prices and rents. The average adjusted R-squares for the price and rent models are 78.4 and 79.3 percent, respectively.\textsuperscript{16} The postcode dummies explain 54.9 and 48.1 percent of the variations in the price and rent regressions, respectively. The next largest contribution is the group of physical characteristics, contributing 9.6 and 12.7 percent to the price and rent variations, respectively. The regression results also show that the percentage of significant coefficients is high, their economic significance is plausible and the directions implied by the estimated coefficients accord with our prior expectations. With some small variations in the exact numbers, these results generally hold separately for each of the 9 yearly regressions. Given this performance, our hedonic approach is expected to control for a large portion of the quality difference between sold and rented houses.\textsuperscript{17}

\textbf{Insert Table 3 Here}

With regard to the regression results of the HM2–HM8 models, the explanatory power of these models falls as less characteristics are included (as expected), with the smallest model, HM8, explaining 63.9 and 62.8 per cent of the variation in prices and rents, respectively (see Table 4). Around 94.0 per cent of the signs of the estimated coefficients remain the same as the corresponding coefficients of the HM1 model. The premiums to an additional bedroom or bathroom or more land area are in most cases in HM2–HM7 higher than those found in the HM1 model. This is expected because the estimated coefficients in the HM2–HM7 models include a positive effect of the omitted characteristics. In summary, we find the performance of the HM2–HM8 models is stable across years and is as expected in relation to the HM1 model.

\textsuperscript{16}The lowest adjusted R-square is 72.1 percent for the price model and 76.5 percent for the rent model. Our adjusted R-squares for the rent models are much larger than those reported by Arévalo and Ruiz-Castillo (2006), Crone et al. (2009) and Kurz and Hoffmann (2009).

\textsuperscript{17}The models do not include interactions between number of bedrooms and number of bathrooms, since the inclusion of interactions between pairs of discrete variables would create problems when calculating our quality-adjusted price-rent ratios. Our hedonic approach requires that both the price and rent models are specified on the same set of characteristics. If a particular combination of characteristics, say 3 bedrooms and 2 bathrooms, is explicitly included in the hedonic models in the form of a dummy variable, then our approach requires that this combination is observed in both the sold and rental data. In many cases, the matching of characteristics at such a level of detail is not observed.
3.2 Adjustments for omitted variables bias in our hedonic models

Our distributions of quality-adjusted price-rent ratios, from which medians can then be calculated, are obtained by bringing together the price-rent ratios from our 8 models (HM1, HM2, . . . , HM8). However, as is explained in section 2.3, a different omitted variables adjustment is made to the imputed price-rent ratios of each model, prior to their pooling into a single data set.

A point of reference is provided by address-matched price-rent ratios, which directly control for quality differences. We have 42,153 dwellings in our data set for which we observe both prices and rents. We have a total of 49,388 selling prices for these dwellings (13.3 percent of the sold data) and 71,566 rents (20.0 percent of the rented data), respectively.\(^{18}\) As shown in (15) and (16), the matching of time periods is attained by extrapolating the prices and rents over time (both backwards and forwards) using price and rent indexes.\(^{19}\) The number of houses which are sold and rented more than once within the sample period are 38,612 and 81,017, respectively (corresponding to 81,568 price and 217,575 rent observations).

Table 5-column 2 provides estimates of the omitted variables bias of the price-rent ratios derived from the HM8 model [i.e., \(\lambda_{t,\text{HM8}}\) derived from (18)]. Conforming to our expectations, we find that for every year \(\lambda_{t,\text{HM8}} > 1\). The average \(\lambda_{t,\text{HM8}}\) for 9 years is 1.115, implying that HM8 models fail to fully adjust for the quality difference between the sold and rented dwellings and, as a result, the price-rent ratios obtained from the HM8 model are on average 11.5 percent higher than those obtained from the address-matched model. Table 5 also provides estimates (see columns 3-9) of \(\lambda_{t,\text{HMj|HM8}}\) in (19) for \(j = 1, \ldots, 7\). Conforming to our expectations, these estimates are less than 1 (with only a few exceptions for individual years). This provides strong support for our hypothesis that sold dwellings perform better than the rented dwellings on the omitted variables. A model with more explanatory variables

\(^{18}\)The number of observations is greater than the number of dwellings because of repeat-sales and repeat-rents. Only around 1500 of these matched houses were sold and rented in the same quarter.

\(^{19}\)The average time span over which prices and rents are extrapolated is 2 and a quarter years, with 90 percent of the extrapolation done for less than 6 years (the larger the time span the less reliable is the extrapolation). Harding, Rosenthal and Sirmans (2007) report that the median time between two sales was 5 years for US data.
has less omitted variables and hence on average lower price-rent ratios.

Insert Table 5 Here

The overall omitted variables bias $\lambda_{t,HMj}$ of model HMj is obtained by multiplying $\lambda_{t,HM8}$ by $\lambda_{t,HMj|HM8}$, as shown in (20). The estimates of $\lambda_{t,HMj}$ are broadly consistent with the inequalities in (21). While there are some slight inconsistencies for individual years, the average results for each model correspond exactly with (21).

3.3 Quality-adjustment bias in price-rent ratios

Three sets of median price-rent ratios for each year in our data set are presented in Table 6. The first series gives the raw quality-unadjusted price-rent ratios. The second series (quality-adjusted 1) gives the price-rent ratios adjusted for characteristics in our data set derived from (14). The third series (quality-adjusted 2) also accounts for missing characteristics in hedonic models HM2-HM8 and omitted variables. These adjustments are made with reference to the subset of dwellings in our data set that both sell and rent.

The raw quality-unadjusted price-rent ratios are on average 18.4 percent larger than their quality-adjusted counterparts (quality-adjusted 2). From Table 6 it can be seen that slightly under half of this (i.e., 8.7 percent) is attributable to differences in the observable characteristics of sold and rented dwellings. The remainder, 9.7 percent, can be attributed to a combination of missing and omitted characteristics. The relative contributions of each of these can be calculated from the adjustment factor of HM1 in Table 5. On average this adjustment factor is $1.115 \times 0.952 = 1.062$, or 6.2 percent, all of which is omitted variables bias since HM1 is the full hedonic model with no missing characteristics. It follows therefore that the remaining 3.5 percent is attributable to missing characteristics.

The average quality difference between owner-occupied and rented dwellings will be even larger than that between sold and rented dwellings, since some fraction of sold dwellings are subsequently rented. Suppose 70 percent of the dwellings sold in our data set are owner-occupied and 30 percent are rented (as is the case on average in Australia). An estimate of the average quality difference between owner-occupied and rented dwellings is 26.3 percent – $(118.4 − 0.3 \times 100)/0.7 = 126.3$.

Insert Table 6 Here

It is noticeable that the magnitude of the quality bias falls towards the end of our sample.
This pattern can be seen clearly in Table 6 and Figure 1 (as the three curves converge in 2008 and 2009). This seems to be due to a fall in the average quality of dwellings sold after the boom ends in 2004, perhaps due to an increase in the number of distressed sales. Focusing on location as a measure of quality, we find that relatively more houses were sold in cheaper postcodes in the latter part of our sample than in the earlier part. More specifically, over the 2001-5 period, 52.9 percent of sold houses were in the cheapest 4 postcode deciles and 30.8 percent were in the most expensive 4 postcode deciles, whereas over the 2006-9 period 58.4 percent of sold houses were in the cheapest 4 deciles and 25.7 percent in the most expensive 4 deciles. In 2009 this postcode distribution shifted even further to 60.7 and 23.1 percent, respectively. By contrast, the locational distribution of rental dwellings remained relatively stable at around 58.0 percent in the cheapest 4 deciles and 27.0 percent in the most expensive 4 deciles over the entire sample period.

3.4 Movement of ratios of price and rent indexes

Figure 2 shows the price and rent indexes obtained from the median, hedonic and repeats methodologies for the whole, lower and upper end of the market. The lower and upper end are defined here as the bottom and top 4 deciles of postcodes, respectively, where these postcodes are ordered from the cheapest to the most expensive, separately for each year.

The three methodologies reveal some common themes. It can be seen that prices rose faster at the lower end during the boom (ending in 2004), after which they fell again. By contrast, at the upper end prices continued to rise after 2004. The movements of rents at the upper and lower end are more synchronized. Both rose throughout the sample, although at a faster rate at the lower end of the market.

Figure 2 highlights the potential distortions that can arise from using price and rent indexes to measure changes in the price-rent ratio, as is done frequently in the literature (see the discussion in the Introduction). For example, dividing a hedonic price index by a repeat-rent index, or a hedonic price index defined on the upper end of the market by a hedonic rent index defined on the lower end, may generate a distorted price-rent ratio series.

Figure 2 also sheds light on the convergence/divergence trend of the cross-section price-
rent ratios. The variance of the log of the price-rent ratio in each quarter is graphed in Figure 3.\textsuperscript{20} The dispersion of the price-rent ratios is U-shaped, with the minimum dispersion being observed in 2004. This corresponds to the peak of the boom in Sydney house prices. The U-shape in Figure 3 (i.e., $\sigma$ convergence followed by later divergence) is explained by the 1993-2004 housing boom in Sydney which started at the top-end of the market (triggered by strong income growth at the top end and the scarcity of dwellings in prime locations) and then gradually rippled down to the low end. Towards the end of the boom, the prices of lower quality dwellings rose faster than those at the top end thus causing price convergence. Price rises at the low end however were probably driven more by momentum than genuine scarcity. Also, buyers at the low end tended to have higher loan-to-value ratios. Hence when the boom ended prices fell at the low end, triggered partly by distressed sales, thus generating the subsequent price divergence.\textsuperscript{21} Meanwhile, the standard deviation of rents over this period was relatively stable. Combining these strands, it follows that price-rent ratios, like prices, first converged and then diverged, generating the U-shaped curve in Figure 3.

\textbf{Insert Figure 3 Here}

4 Detection of Departures from Equilibrium

4.1 The user-cost equilibrium condition

The user cost of a durable good is the present value of buying it, using it for one period and then selling it (see Hicks 1946). In equilibrium this should equal the cost of renting the good for one period. Following Himmelberg et al. (2005) and Girouard et al. (2006), the equilibrium condition can be written as follows:

\begin{equation}
R_t = u_t P_t,
\end{equation}

where $R_t$ is the period $t$ rental price, $P_t$ the purchase price, $u_t P_t$ is user cost, and $u_t$ the per dollar user cost. In a housing context, per dollar user cost can be calculated as follows:

\begin{equation}
u_t = r_t + \omega_t + \delta_t + \gamma_t - g_t,
\end{equation}

\textsuperscript{20}The variance of the log of the price-rent ratio is scaled up by a factor of 10 to fit in the Figure with the variances of price and rent.

\textsuperscript{21}This perhaps explains why, as is discussed in the previous section, the proportion of sales from cheaper postcodes rose after the boom ended.
where \( r \) denotes the risk-free interest rate, \( \omega \) is the property tax rate, \( \delta \) the depreciation rate for housing, \( \gamma \) the risk premium of owning as opposed to renting, and \( g \) the expected capital gain. That is, an owner occupier foregoes interest on the market value of the dwelling, incurs property taxes and depreciation, incurs risk (mainly due to the inherent uncertainty of future price and rent movements in the housing market) and benefits from any capital gains on the dwelling.\(^{22}\) If \( R_t > u_t P_t \), owner-occupying becomes more attractive and hence this should exert upward pressure on \( P \) and downward pressure on \( R \) until equilibrium is restored. The converse argument applies when \( R_t < u_t P_t \).

Rearranging (24), we obtain that in equilibrium the price-rent ratio should equal the reciprocal of per dollar user cost (i.e., \( P_t/R_t = 1/u_t \)). If the actual price-rent ratio exceeds, or is less than, our estimate of the reciprocal of per dollar user cost it follows that the housing market is not in equilibrium.

Such departures can happen due to transaction costs, time lags for building, variations over time in mortgage lending standards, and the extrapolative component of expected capital gains. Together these factors may cause prices to overshoot in booms and undershoot in busts (see Abraham and Hendershott 1996).

The equilibrium condition (24), however, implicitly assumes that \( P_t \) and \( R_t \) are calculated for properties of equivalent quality. If instead \( P_t \) refers to dwellings which are of superior quality to the dwellings referred by \( R_t \), then \( P_t/R_t \) is overestimated and, as a result, the user cost equilibrium condition will be biased towards finding that the price-rent ratio is above its equilibrium level.

A second problem with the equilibrium condition is that the expected capital gain \( g \) is not directly observable. \( g \) can be separated into two components: the expected real capital gain and expected inflation. Of these, the expected real capital gain is more problematic. A standard approach is to assume that the expected real capital gain is extrapolated from the past performance of the housing market. This, however, may cause \( 1/u \) to fluctuate a lot over time, thus potentially seriously undermining its usefulness (see Verbrugge 2008 and Diewert 2009). Extrapolation may also push up the equilibrium price-rent ratio during a boom, thus

\(^{22}\)In some countries, owner-occupiers get the benefit from the tax deduction on the mortgage interest payments (see Girouard et al. 2006 for a list of OECD countries providing such benefits). For these countries, \( r_t \) should be adjusted to include the offsetting tax benefit. However, no such benefit is provided to the owner occupiers in Australia.
potentially undermining the user-cost equilibrium condition’s ability to detect a departure from equilibrium. We explore in the next section the impact of alternative extrapolation time horizons.

Alternatively, the expected capital gain can be imputed from the equilibrium user cost condition. To the best of our knowledge, this approach – first suggested by Diewert (1983) – has never before been applied to data. This is perhaps because it requires price-rent ratios in levels which are difficult to obtain. More specifically, rearranging the user cost formula in (25) and imposing the equilibrium condition in (24) yields the following:

\[ g_t = r_t + \omega_t + \delta_t + \gamma_t - \frac{R_t}{P_t}. \]  

(26)

Setting \( R_t/P_t \) equal to the reciprocal of the median quality-adjusted price-rent ratio in period \( t \) and inserting estimates of \( r_t, \omega_t, \delta_t \) and \( \gamma_t \), we obtain an estimate of \( g_t \). If the resulting capital gain \( g_t \) is deemed implausible, then we can conclude that the assumption of equilibrium is incorrect.

We can quantify the impact of quality bias on our perception of the housing market by substituting the quality-unadjusted price-rent ratio \([R_t/P_t]_{\text{unadj}}\) in place of the quality-adjusted price-rent ratio \([R_t/P_t]_{\text{adj}}\) in (26) as follows:

\[ G_t = \left[ \frac{R_t}{P_t}_{\text{adj}} \right] - \left[ \frac{R_t}{P_t}_{\text{unadj}} \right]. \]  

(27)

\( G_t \), which does not depend on the chosen values of all the parameters in (26), measures how quality bias affects the expected capital gain required for equilibrium. We apply this approach to our Sydney data in section 4.3.

### 4.2 Equilibrium versus actual price-rent ratios

The values we use for the variables in the user cost equation in (25) are shown in Table 7.

\( r \) is the 10-year interest rate on Australian government bonds.\(^{23}\) Our use of the 10-year interest rate on the loan-to-value ratio of purchasers. The relevant interest rate for a purchaser with a 100 percent loan-to-value ratio is the mortgage interest rate \( r^M \), while for a purchaser with a 0 percent loan-to-value ratio it is the risk-free 1-year rate \( r^{rf} \). According to Green and Wachter (2005, Table 2), the average loan-to-value ratio in Australia is 63 percent. Assuming this figure remains constant, we could calculate \( r \) as follows: \( r^* = 0.37 \times r^{rf} + 0.63 \times r^M \). Interestingly, over our sample, \( r^* \) and the 10-year interest rate are quite similar. On average \( r^* \) is 0.09 percentage points higher. It follows that the choice between using the 10-year government bond rate and \( r^* \) has virtually no impact on our results.

\(^{23}\)Alternatively, we could have used the mortgage interest rate. Whether this is appropriate depends on the loan-to-value ratio of purchasers. The relevant interest rate for a purchaser with a 100 percent loan-to-value ratio is the mortgage interest rate \( r^M \), while for a purchaser with a 0 percent loan-to-value ratio it is the risk-free 1-year rate \( r^{rf} \). According to Green and Wachter (2005, Table 2), the average loan-to-value ratio in Australia is 63 percent. Assuming this figure remains constant, we could calculate \( r \) as follows: \( r^* = 0.37 \times r^{rf} + 0.63 \times r^M \). Interestingly, over our sample, \( r^* \) and the 10-year interest rate are quite similar. On average \( r^* \) is 0.09 percentage points higher. It follows that the choice between using the 10-year government bond rate and \( r^* \) has virtually no impact on our results.
rate rather than a 1-year rate can be justified as follows:

Looking at the opportunity costs of owning a house from the viewpoint of an owner occupier, the relevant time horizon . . . is the expected time the owner expects to use the dwelling before reselling it. This time horizon is typically some number between six and twelve years. (Diewert 2009, p. 494).

The 10-year government bond rate remained reasonably stable over the 2001-9 period, ranging between a minimum value of 5.0 percent in 2009 and a maximum value of 6.0 percent in 2007. (Source: Reserve Bank of Australia)

\[ \omega = 1.0 \, \text{percent.} \] This is an estimate for an average land tax over the 2001-2009 period. (Source: Office of State Revenue, New South Wales, Australia)

\[ \delta = 2.5 \, \text{percent.} \] This is the gross depreciation rate estimated by Harding, Rosenthal and Sirmans (2007) using American Housing Survey data over the period 1983 to 2001. This is also the rate used by Himmelberg et al. (2005).

\[ \gamma = 2.0 \, \text{percent.} \] This is the risk premium estimated by Flavin and Yamashita (2002) and used by Himmelberg et al. (2005).\(^{24}\)

It should be noted that Girouard et al. (2006) fix \(\delta + \gamma\) at 4 percent for the 18 OECD countries (including Australia) they studied over the 1990-2004 period. Verbrugge (2008) fixes \(\omega + \delta + \gamma\) at 7 percent for the US over the 1980-2004 period. The values of our parameters lie in between these estimates. (The higher the values of these parameters, the more likely it is that the price-rent will be found to be above its equilibrium level.)

\(g\) is the expected nominal capital gain which consists of the sum of the expected real capital gain and expected inflation.\(^{25}\) The expected real capital gain in year \(t\) is assumed to equal the moving average of real capital gain over the preceding \(x\) years. We consider three

\(^{24}\)The high price volatility relative to rent volatility in housing, implies a higher level of risk associated with home purchases. Han (2010) reports that the home price risk is one of the most significant risks that homeowners in the U.S. face. In our data set, we find that the standard deviation of the log of prices is higher than the standard deviation of the log of rents in every year and on average 20 percent higher over the whole sample period.

\(^{25}\)We need to separate expected real capital gains from inflation due to the change over time in the inflation environment in Australia. From 1981-1990 the average inflation rate in Sydney (as measured by the consumer price index) was 8 percent. By contrast, it was 2.6 percent from 1990-2000, and 3.1 percent from 2000-2009. The expected nominal capital gain on housing in the 1980s (most of which is inflation) must therefore have been much higher than in the 1990s and 2000s.
different values of $x$ (i.e., 10, 20 and 30 years). More precisely, the expected capital gain in year $t$ is calculated as follows:

$$
\text{Expected real capital gain}_t = \left( \frac{EHPI_t/CPI_t}{EHPI_{t-x}/CPI_{t-x}} \right)^{1/x}.
$$

Here $EHPI_t$ is the level of the Established House Price Index and $CPI_t$ is the level of the consumer price index for Sydney in year $t$. Both the EHPI and CPI are computed by the Australian Bureau of Statistics (ABS).\textsuperscript{26,27} For example, for $x=20$ years, the expected annual real capital gain ranges from a peak of 4.8 percent in 2004 to a low of 1.8 percent in 2009 (see Table 7).

The expected rate of inflation is assumed to be 3 percent. This is very close to the average rate of inflation over the 2001-9 period which equalled 3.07 percent. It is also the upper bound on the Reserve Bank of Australia’s inflation target (which is 2-3 percent).

Insert Table 7 Here

Inserting these values into (25) yields the values shown in Table 7 for the equilibrium price-rent ratio $1/u_t$ each year. The assumed time horizon of past performance over which expected capital gains are calculated plays a pivotal role. When the time horizon is 30 years, the equilibrium price-rent ratio ranges between 17.6 and 22.4. Over a 20 year horizon the range rises to between 17.4 and 32.0, while over a 10 year horizon the range is from 19.2 to 53.0. If the time horizon is reduced to 5 years, then in some years the equilibrium price-rent ratio rises to infinity since the expected capital gain is large enough to make the user cost become negative.

The extreme volatility of per dollar user cost when expected capital gains are extrapolated from past performance over short time horizons has been noted previously by Verbrugge (2008) and Diewert (2007, 2009). Diewert (2009), citing evidence on the length of housing booms and busts from Girouard et al. (2006), argues that a longer time horizon (between 10

\textsuperscript{26}The Established House Price Index (EHPI) only goes back to 1986. To obtain prices back to 1981 or 1971 (for the cases where $x=20$ or 30), the EHPI was spliced together with an index calculated by Abelson and Chung (2005). See Stapledon (2007) for a discussion of why the Abelson and Chung series is probably the best available option for extending the EHPI back before 1986. In addition, the methodology underlying the EHPI changed slightly in 2005. Hence to obtain our full series, it was also necessary to splice together the pre and post 2005 EHPI series.

\textsuperscript{27}The EHPI is computed using the stratified-median approach, which may fail to fully adjust for quality changes over time. Given the EHPI is probably the most widely followed house price index for Sydney, it nevertheless is a useful benchmark for describing expectations of capital gains.
and 20 years) is more plausible in terms of how market participants form their expectations.

In the case of Sydney, even 20 years does not seem to be enough, since the strong performance of the housing market in the years leading up to 2004 pushes up the equilibrium price-rent ratio even more than the actual price-rent ratio. The situation is even worse when the extrapolation time horizon is only 10 years. We are led then to the perverse conclusion that the actual price-rent ratio in 2003-5 is far below its equilibrium level.

We therefore favor extrapolating expected real capital gains over 30 years. In this case, as can be seen in Table 7, we can conclude that while the price-rent ratio was above its equilibrium level from 2001 to 2008 (by nearly 50 percent in 2004), this was no longer true in 2009. The market has gone through a gradual correction process since 2004. This can be attributed to the combination of stable or falling prices since 2004 accompanied by a steady rise in rents leading to a gradual fall in the price-rent ratio (see Figure 2) and a fall in the 10-year interest rate (see Table 7).

Over our whole sample, the quality-adjusted price-rent ratio is on average 4.3 percentage points above its equilibrium level. This difference rises to 8.3 percentage points in the absence of quality adjustment. Hence quality adjustment essentially halves the magnitude of the measured departure from equilibrium.

4.3 Imputing expected capital gains from the user-cost equilibrium condition

Substituting the values for \( r_t, \omega, \delta, \gamma \) and \( \pi_t^e \) from Table 7 and the quality-adjusted median price-rent ratios from Table 6 into (26), yields the expected real capital gains series shown in Table 7. The implied expected real annual capital gain if the market is in equilibrium exceeds 4 percent from 2002 to 2007, reaching a peak of 4.7 percent in 2004.

By comparison, Gyourko, Mayer and Sinai (2006) find that the average annual real capital gain for the 50 US cities in their sample over the period 1950 to 2000 was 1.7 percent, with the highest result of 3.5 percent being observed for San Francisco. There are a number of similarities between San Francisco and Sydney, ranging from desirable coastal locations and scarcity of land to population growth. In this sense San Francisco is perhaps not a bad benchmark for Sydney.

We can conclude therefore that the price-rent ratio was above its equilibrium level in
Sydney at least from 2002 to 2007. By 2009, however, the implied real expected capital gain in equilibrium had fallen to 2.9 percent, which seems entirely plausible.

In the absence of quality adjustment, the implied expected real capital gain in equilibrium is 5 percent or higher in 2002, 2003, 2004 and 2006. The impact of quality bias on the implied real expected capital gain in equilibrium is measured by $G_t$ in (27). The largest value of $G_t$ (i.e., 1 percentage point) is observed in 2001. On average over our whole sample, the quality-unadjusted implied expected real capital gains are 0.6 percentage points higher than their quality-adjusted counterparts. Again, therefore, failure to quality adjust leads to the conclusion that the departure from equilibrium (particularly from 2002-7) was larger than it actually was.

4.4 Cross-sectional variation of price-rent ratios

Any variation in price-rent ratios across the housing distribution has implications for the detection of departures from equilibrium. We observe that the price-rent ratio increases steadily as we move from the lower to the upper end of the market. By pooling data across years, and by regressing the log of the estimated quality-adjusted price-rent ratio for sold dwellings on the log of prices, we find that the price-rent ratio increases by 0.21 percent for each percent increase in prices, and, similarly, by regressing the log of the quality-adjusted price-rent ratio for rented dwellings on the log of rents, we find that the price-rent ratio increases by 0.11 percent for each percent increase in rents (see Table 8).

Insert Table 8 Here

The same pattern can be seen from a different perspective. Ordering all dwellings sold and rented each year from cheapest to most expensive, we compute a quality-adjusted price-rent ratio for the lower quartile, median and upper quartile sold dwellings and likewise for the lower quartile, median and upper quartile rented dwellings (see Table 9). The quality-adjusted price-rent ratio in Table 9 is lowest for the first quartile, followed by the median, and is highest for the upper quartile. The price-rent ratio for the lower quartile sold dwelling (in terms of its price) is 23.1 while the corresponding price-rent ratio for the upper quartile is 27.17. The results for the lower and upper quartiles from the rental data are similar, i.e., 22.75 and 26.13 respectively.

28 This pattern of a rising price-rent ratio can also be discerned as we move from cheaper to more expensive postcodes.
Insert Table 9 Here

This trend has been previously noted by Heston and Nakamura (2009) and Aten, Figueroa and Martin (2011). Heston and Nakamura for example find that the price-rent ratio rises by more than 50 percent as the price of a dwelling increases from $50,000 to $500,000. Their study uses survey data on four regions in the US (Alaska, the Caribbean, the Pacific and Washington D.C.) for 1990, where the rent and price of an owner-occupied dwelling is self-estimated by owners.

According to the user cost equilibrium condition, per dollar user cost should equal rental yield (i.e., the reciprocal of the price-rent ratio). If rental yield is lower at the top end this therefore implies that either per dollar user cost must be likewise lower at the top end or that the equilibrium condition does not hold for all segments of the market. We consider three explanations that focus on the former and two that focus on the latter.

Taking the latter first, the owner of an expensive dwelling may wish to rent for temporary purposes or to rent to someone reliable who will maintain the property properly. The rent is offered at a discount in order to attract either lower income households or higher income households who would otherwise prefer to owner-occupy (see Diewert 2009). To the extent this is true, it follows that rental yield will be lower than per dollar user cost at the top end of the market. Second, households – particularly those with low incomes or wealth – may be credit constrained. They may prefer to buy than rent, but cannot get a large enough mortgage to do so. This will cause rental yield to be higher than per dollar user cost at the low end of the market (see Duca, Muellbauer and Murphy 2011). As is explained later, these first two explanations have implications for the measurement of GDP.

Alternatively, user cost may be lower (and hence the equilibrium price-rent ratio higher) at the top end for one of the following reasons. First, the depreciation rate should be lower at the top end of the market. Structures depreciate while land does not, and the value of the land relative to the value of the structure is typically higher at the top-end of the market (see Diewert 2009, and Diewert, de Haan and Hendriks 2012). Himmelberg et al. (2005) make a similar argument in the context of comparisons of price-rent ratios across cities. They argue that the high value of land in San Francisco and New York should act to lower the per dollar user cost in these cities. Second, the risk premium may be lower at the top end of the market, thus lowering the per dollar user cost. Some evidence to support this hypothesis is provided in Figure 2(a), where it is shown that prices at the low end of the market rose more at the
end of the boom in Sydney and then fell more as boom turned to bust. Finally, the expected capital gain may be higher at the top end of the market (which again acts to lower the per dollar user cost). While this is unlikely to always be the case, the rise in inequality observed in many countries since the 1970s may be supporting such expectations over a sustained period.

It follows therefore that to obtain a comprehensive picture of the housing market it is necessary to consider the whole price-rent ratio distribution and the way user cost varies in different segments of the market.

5 Some Implications for the Measurement of GDP

Housing services are an important component of GDP. For example, imputed rent of owner-occupied housing accounts for about 8-9 percent of GDP in the US while tenant rent accounts for about 2-3 percent (see Grist 2010). Our findings have highlighted some of the difficulties that can arise when measuring the flow of housing services. In particular, imputing rent for owner-occupied housing by matching characteristics of rented and owner-occupied dwellings could, in the absence of an omitted variables adjustment, cause the service flow from owner-occupied housing to be underestimated. In our case, we find a 13 percent quality difference between rented and owner-occupied dwellings matched on land area, number of bedrooms, number of bathrooms and postcode. Assuming owner-occupied housing’s share in GDP is 8.5 percent, this translates into a downward bias in GDP of over 1 percent (i.e., $0.13 \times 0.085$). With less exact matching of characteristics the bias will be even larger.

Mismatches between per dollar user cost and rental yield may also have implications for GDP. When they are not equal, it is not clear which out of user cost and rent (actual or imputed) should be used to measure the flow of housing services. According to Diewert, Nakamura and Nakamura (2009), the value of housing services equals the maximum of rent and user cost (i.e., the opportunity cost). In this case, our results imply that the current methodology that equates the service flow to rent may overstate GDP during booms (where the price-rent ratio is typically above its equilibrium level and hence per dollar user cost exceeds the rental yield), and understate GDP during busts (where the reverse is true).

Over and above these variations over the business cycle, housing services may be overestimated at the low end (where because of credit constraints rental yield may exceed per
dollar user cost) and underestimated at the top end of the market (where because of a lack of high income renters per dollar user cost may exceed rental yield).  

6 Conclusion

We develop a hedonic approach that imputes price-rent ratios at the level of individual dwelling. These ratios are corrected for omitted variables using a subsample of dwellings that both sell and rent during our sample period.

Applying this approach to a data set consisting of 730,000 price and rent observations for Sydney, Australia over the period 2001-9, we find on average an 18 percent quality difference between sold and rented dwellings, which rises to 26 percent between owner-occupied and rented dwellings. The magnitude of this quality difference, however, varies significantly over time.

Quality adjustment therefore acts to significantly reduce observed price-rent ratios. Even so, we find that price-rent ratios in Sydney were above their equilibrium level (obtained from the user-cost condition) from 2001-8, although this not so in 2009. Without quality adjustment the departure from equilibrium seems even larger than it actually was. We also find that both actual and equilibrium price-rent ratios vary at different points in the housing distribution, thus implying that it is not enough to focus just on the median when assessing whether or not the housing market is out of equilibrium.

References


29 The expansion of the subprime market in the US and other countries in the decade leading up to 2008 may have narrowed the gap between rental yield and per dollar user cost at the low end of the market by allowing more low wealth households to switch from renting to owner-occupying. An implication of the Diewert-Nakamura-Nakamura approach is that failure to account for this trend will impart an upward bias to the growth rate of GDP. The direction of the bias should have reversed since 2008 with the contraction of the subprime market.
ABS (2010), Measures of Australia’s Progress, 2010, Cat. No. 1370.0. Available at: http://www.abs.gov.au/ausstats/abs@.nsf/2f762f95845417ae25706c00834efa/8cba7b65667c8d59ca25779c001c481d


Kurz, C. and J. Hoffmann (2009), “A Rental Equivalence Index for Owner-Occupied Housing in West Germany,” in Diewert, W. E., B. M. Balk, D. Fixler, K. J. Fox and A. O.


<table>
<thead>
<tr>
<th>Statistics</th>
<th>Price Data</th>
<th>Rental Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of observations</td>
<td>371,604</td>
<td>358,381</td>
</tr>
<tr>
<td>Period of coverage (years)</td>
<td>9 (2001–09)</td>
<td>9 (2001–09)</td>
</tr>
<tr>
<td>Median price or annual rent (AU $)</td>
<td>495,000.00</td>
<td>18,250.00</td>
</tr>
<tr>
<td>Median land area (square meters)</td>
<td>592.00</td>
<td>584.00</td>
</tr>
<tr>
<td>Mean land area (square meters)</td>
<td>684.39 (568.54)</td>
<td>640.86 (390.11)</td>
</tr>
<tr>
<td>Median no. of bedrooms</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Mean no. of bedrooms</td>
<td>3.31 (0.84)</td>
<td>3.17 (0.82)</td>
</tr>
<tr>
<td>Modal no. of bedrooms</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Median no. of bathrooms</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Mean no. of bathrooms</td>
<td>1.68 (0.74)</td>
<td>1.45 (0.64)</td>
</tr>
<tr>
<td>Modal no. of bathrooms</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Percentage of observations having the following characteristics:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Land area, no. of bedrooms and no. of bathrooms</td>
<td>61.67</td>
<td>45.60</td>
</tr>
<tr>
<td>No. of bedrooms and no. of bathrooms</td>
<td>62.43</td>
<td>49.33</td>
</tr>
<tr>
<td>Land area and no. of bathrooms</td>
<td>61.67</td>
<td>45.60</td>
</tr>
<tr>
<td>Land area and no. of bedrooms</td>
<td>73.50</td>
<td>47.97</td>
</tr>
<tr>
<td>No. of bathrooms</td>
<td>62.43</td>
<td>49.33</td>
</tr>
<tr>
<td>No. of bedrooms</td>
<td>74.46</td>
<td>99.95</td>
</tr>
<tr>
<td>Land area</td>
<td>98.43</td>
<td>47.99</td>
</tr>
</tbody>
</table>

Note: The figures in the parentheses are the estimated standard errors.
### Table 2: Distributions of Characteristics in the Price and Rental Data (in %)

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Data</th>
<th>Counts</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Bedrooms</strong></td>
<td>Price</td>
<td>0.62</td>
<td>13.30</td>
<td>49.40</td>
<td>29.22</td>
<td>6.48</td>
<td>0.98</td>
<td>100.00</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Rent</td>
<td>1.52</td>
<td>15.48</td>
<td>53.22</td>
<td>23.79</td>
<td>5.88</td>
<td>0.11</td>
<td>100.00</td>
<td></td>
</tr>
<tr>
<td><strong>Bathrooms</strong></td>
<td>Price</td>
<td>46.74</td>
<td>40.07</td>
<td>11.61</td>
<td>1.40</td>
<td>0.18</td>
<td>n.a.</td>
<td>100.00</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Rent</td>
<td>61.74</td>
<td>31.54</td>
<td>6.20</td>
<td>0.48</td>
<td>0.03</td>
<td>n.a.</td>
<td>100.00</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Data</th>
<th>Deciles</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
<th>5th</th>
<th>6th</th>
<th>7th</th>
<th>8th</th>
<th>9th</th>
<th>10th</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Postcodes</strong></td>
<td>Price</td>
<td>17.62</td>
<td>14.91</td>
<td>11.79</td>
<td>8.30</td>
<td>9.66</td>
<td>8.28</td>
<td>8.97</td>
<td>8.09</td>
<td>7.33</td>
<td>5.04</td>
<td></td>
</tr>
<tr>
<td>(by Price)†</td>
<td>Rent</td>
<td>20.40</td>
<td>15.73</td>
<td>12.35</td>
<td>7.16</td>
<td>8.98</td>
<td>8.09</td>
<td>7.97</td>
<td>7.65</td>
<td>7.15</td>
<td>4.52</td>
<td></td>
</tr>
<tr>
<td><strong>Postcodes</strong></td>
<td>Price</td>
<td>18.56</td>
<td>15.46</td>
<td>9.31</td>
<td>7.60</td>
<td>10.96</td>
<td>9.40</td>
<td>9.22</td>
<td>7.91</td>
<td>6.46</td>
<td>5.11</td>
<td></td>
</tr>
<tr>
<td>(by Rent)§</td>
<td>Rent</td>
<td>19.98</td>
<td>16.97</td>
<td>10.07</td>
<td>7.27</td>
<td>10.55</td>
<td>8.58</td>
<td>8.46</td>
<td>7.69</td>
<td>5.92</td>
<td>4.50</td>
<td></td>
</tr>
</tbody>
</table>

*The price and rental data are pooled before dividing them into deciles in terms of land area. Therefore, each decile corresponds to the same land area in both data sets.
†Houses are ordered from the cheapest to the most expensive in terms of price.
§Houses are ordered from the cheapest to the most expensive in terms of rent.

### Table 3: HM1 Regression Results

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Price Models</th>
<th>Rent Models</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of observations</td>
<td>25462 (12273)</td>
<td>18157 (16235)</td>
</tr>
<tr>
<td>No. of parameters</td>
<td>204 (17)</td>
<td>204 (17)</td>
</tr>
<tr>
<td>Adjusted $R^2$ (%)</td>
<td>78.35 (3.29)</td>
<td>79.33 (1.95)</td>
</tr>
<tr>
<td>Location attributes:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Joint contribution (%)</td>
<td>54.9 (2.89)</td>
<td>48.08 (1.94)</td>
</tr>
<tr>
<td>% of significant coefficients</td>
<td>93.26 (2.10)</td>
<td>85.77 (7.53)</td>
</tr>
<tr>
<td>Temporal attributes:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Joint contribution (%)</td>
<td>0.32 (0.33)</td>
<td>0.13 (0.11)</td>
</tr>
<tr>
<td>% of significant coefficients</td>
<td>96.30 (11.11)</td>
<td>55.56 (52.70)</td>
</tr>
<tr>
<td>Physical attributes:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Joint contribution (%)</td>
<td>9.60 (0.93)</td>
<td>12.74 (3.76)</td>
</tr>
<tr>
<td>% of significant coefficients</td>
<td>84.13 (8.58)</td>
<td>68.25 (8.07)</td>
</tr>
</tbody>
</table>

Note: The numbers are the mean results obtained from the 9 yearly regressions. The numbers in parentheses are the standard deviations of the 9 yearly regressions, indicating how stable or dispersed the statistics are across yearly regressions. The joint contribution is calculated by taking the difference in the adjusted $R^2$ between the unrestricted and restricted models. The statistical tests are conducted at the 5% significance level.
Table 4: HM2–HM8 Regression Results

<table>
<thead>
<tr>
<th>Statistics</th>
<th>HM2</th>
<th>HM3</th>
<th>HM4</th>
<th>HM5</th>
<th>HM6</th>
<th>HM7</th>
<th>HM8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adjusted $R^2$ (in %)</td>
<td>Price</td>
<td>75.92</td>
<td>76.67</td>
<td>74.90</td>
<td>74.19</td>
<td>72.36</td>
<td>67.06</td>
</tr>
<tr>
<td></td>
<td>Rent</td>
<td>78.54</td>
<td>74.44</td>
<td>75.18</td>
<td>73.97</td>
<td>72.50</td>
<td>65.85</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(4.55)</td>
<td>(2.71)</td>
<td>(3.65)</td>
<td>(3.86)</td>
<td>(5.09)</td>
<td>(2.87)</td>
</tr>
<tr>
<td>% of coefficients having</td>
<td>Price</td>
<td>92.29</td>
<td>98.76</td>
<td>99.20</td>
<td>92.26</td>
<td>91.15</td>
<td>97.32</td>
</tr>
<tr>
<td>the same sign as HM1</td>
<td></td>
<td>(5.43)</td>
<td>(1.17)</td>
<td>(6.64)</td>
<td>(5.18)</td>
<td>(6.06)</td>
<td>(1.81)</td>
</tr>
<tr>
<td></td>
<td>Rent</td>
<td>97.44</td>
<td>95.80</td>
<td>97.04</td>
<td>95.52</td>
<td>92.48</td>
<td>91.37</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2.15)</td>
<td>(1.99)</td>
<td>(1.89)</td>
<td>(4.35)</td>
<td>(2.96)</td>
<td>(3.06)</td>
</tr>
</tbody>
</table>

Notes: (1) The numbers are the mean results obtained from the 9 yearly regressions. The numbers in parentheses are the standard deviations of the 9 yearly regressions.
(2) HM1–HM8 regressions include the following physical characteristics and their interaction terms. HM1: bed, bath, land; HM2: bed, bath; HM3: bath, land; HM4: bed, land; HM5: bath; HM6: bed; HM7: land; HM8: no physical characteristics.

Table 5: Omitted Variables Adjustment Factors: $\lambda_{t,HM8}$ and $\lambda_{t,HMJ|HM8}$

| Year | $\lambda_{t,HM8}$ | $\lambda_{t,HMJ|HM8}, j = 1, \ldots, 8$ |
|------|-------------------|---------------------------------|
|      |                   | HM1    | HM2    | HM3    | HM4    | HM5    | HM6    | HM7    | HM8    |
| 2001 | 1.160             | 0.943  | 0.946  | 0.944  | 0.981  | 0.944  | 0.981  | 0.989  | 1.000  |
| 2002 | 1.169             | 0.948  | 0.951  | 0.947  | 0.980  | 0.948  | 0.987  | 0.993  | 1.000  |
| 2003 | 1.120             | 0.943  | 0.945  | 0.944  | 0.977  | 0.948  | 0.981  | 0.993  | 1.000  |
| 2004 | 1.115             | 0.939  | 0.953  | 0.939  | 0.975  | 0.953  | 0.987  | 0.992  | 1.000  |
| 2005 | 1.103             | 0.962  | 0.967  | 0.960  | 0.987  | 0.962  | 0.992  | 0.999  | 1.000  |
| 2006 | 1.121             | 0.960  | 0.963  | 0.962  | 0.987  | 0.963  | 0.991  | 0.995  | 1.000  |
| 2007 | 1.117             | 0.954  | 0.949  | 0.966  | 0.975  | 0.960  | 0.974  | 1.002  | 1.000  |
| 2008 | 1.084             | 0.956  | 0.956  | 0.966  | 0.973  | 0.964  | 0.976  | 1.005  | 1.000  |
| 2009 | 1.052             | 0.965  | 0.960  | 0.972  | 0.980  | 0.968  | 0.984  | 1.004  | 1.000  |
| Average | 1.115     | 0.952  | 0.954  | 0.955  | 0.979  | 0.957  | 0.984  | 0.997  | 1.000  |

Note: Overall adjustment factor: $\lambda_{t,HMJ} = \lambda_{t,HM8} \times \lambda_{t,HMJ|HM8}$. For example, $\lambda_{2001,HM1} = 1.160 \times 0.943 = 1.094$. 

38
Table 6: Actual and Quality-Adjusted Median Price-Rent Ratios and Quality Bias

<table>
<thead>
<tr>
<th>Year</th>
<th>Actual</th>
<th>Quality Unadjusted</th>
<th>Quality Adjusted 1</th>
<th>Quality Adjusted 2</th>
<th>Bias 1 (%)</th>
<th>Bias 2 (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2001</td>
<td>25.63</td>
<td>23.20</td>
<td>20.48</td>
<td>10.46</td>
<td>25.15</td>
<td></td>
</tr>
<tr>
<td>2003</td>
<td>34.72</td>
<td>32.03</td>
<td>28.59</td>
<td>8.38</td>
<td>21.45</td>
<td></td>
</tr>
<tr>
<td>2004</td>
<td>35.48</td>
<td>32.33</td>
<td>29.78</td>
<td>9.74</td>
<td>19.13</td>
<td></td>
</tr>
<tr>
<td>2005</td>
<td>33.41</td>
<td>29.55</td>
<td>27.09</td>
<td>13.11</td>
<td>23.32</td>
<td></td>
</tr>
<tr>
<td>2006</td>
<td>32.06</td>
<td>27.79</td>
<td>25.44</td>
<td>15.36</td>
<td>26.01</td>
<td></td>
</tr>
<tr>
<td>2007</td>
<td>27.45</td>
<td>25.13</td>
<td>23.60</td>
<td>9.23</td>
<td>16.33</td>
<td></td>
</tr>
<tr>
<td>2008</td>
<td>23.01</td>
<td>22.22</td>
<td>21.40</td>
<td>3.55</td>
<td>7.52</td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>29.34</td>
<td>26.96</td>
<td>24.70</td>
<td>8.70</td>
<td>18.36</td>
<td></td>
</tr>
</tbody>
</table>

Notes: (1) The quality-adjusted 1 series is calculated from (14). It quality adjusts based on the characteristics available in our data set. The quality-adjusted 2 series also accounts for missing characteristics in some of our hedonic models and omitted variables (with the latter adjustments made with reference to the subset of dwellings in our data set that both sell and rent). (2) Bias k = 100 × [(Actual/Quality-adjusted k) - 1].

Table 7: A Comparison of Equilibrium and Actual Price-Rent Ratios

<table>
<thead>
<tr>
<th>Year</th>
<th>$r_t$</th>
<th>$g_t - \pi^e_t$</th>
<th>Equilibrium $P_t/R_t$</th>
<th>Actual $P_t/R_t$</th>
<th>Implied $g_t - \pi^e_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$x=10$</td>
<td>$x=20$</td>
<td>$x=30$</td>
<td>$x=10$</td>
<td>$x=20$</td>
</tr>
<tr>
<td>2001</td>
<td>0.056</td>
<td>0.029</td>
<td>0.024</td>
<td>0.024</td>
<td>19.18</td>
</tr>
<tr>
<td>2002</td>
<td>0.058</td>
<td>0.045</td>
<td>0.037</td>
<td>0.028</td>
<td>26.11</td>
</tr>
<tr>
<td>2003</td>
<td>0.054</td>
<td>0.060</td>
<td>0.047</td>
<td>0.031</td>
<td>52.88</td>
</tr>
<tr>
<td>2004</td>
<td>0.056</td>
<td>0.062</td>
<td>0.048</td>
<td>0.033</td>
<td>52.97</td>
</tr>
<tr>
<td>2005</td>
<td>0.053</td>
<td>0.059</td>
<td>0.047</td>
<td>0.033</td>
<td>47.57</td>
</tr>
<tr>
<td>2006</td>
<td>0.056</td>
<td>0.056</td>
<td>0.043</td>
<td>0.033</td>
<td>38.47</td>
</tr>
<tr>
<td>2007</td>
<td>0.060</td>
<td>0.053</td>
<td>0.045</td>
<td>0.036</td>
<td>32.41</td>
</tr>
<tr>
<td>2008</td>
<td>0.058</td>
<td>0.048</td>
<td>0.042</td>
<td>0.033</td>
<td>24.66</td>
</tr>
<tr>
<td>2009</td>
<td>0.050</td>
<td>0.034</td>
<td>0.018</td>
<td>0.030</td>
<td>26.14</td>
</tr>
<tr>
<td>Mean</td>
<td>0.056</td>
<td>0.049</td>
<td>0.038</td>
<td>0.031</td>
<td>35.60</td>
</tr>
</tbody>
</table>

Notes: $\omega_t = 0.01$, $\delta_t = 0.025$, $\gamma_t = 0.02$ and $\pi^e_t = 0.03$ for the entire period.
Table 8: Cross-Sectional Variation of Price-Rent Ratios

<table>
<thead>
<tr>
<th></th>
<th>2001</th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
<th>2008</th>
<th>2009</th>
<th>Pooled</th>
</tr>
</thead>
<tbody>
<tr>
<td>Δ in price</td>
<td>0.213</td>
<td>0.200</td>
<td>0.181</td>
<td>0.140</td>
<td>0.134</td>
<td>0.145</td>
<td>0.188</td>
<td>0.211</td>
<td>0.213</td>
<td>0.206</td>
</tr>
<tr>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.002)</td>
<td>(0.002)</td>
<td>(0.002)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>Δ in rent</td>
<td>0.201</td>
<td>0.171</td>
<td>0.100</td>
<td>0.077</td>
<td>0.106</td>
<td>0.144</td>
<td>0.187</td>
<td>0.209</td>
<td>0.225</td>
<td>0.111</td>
</tr>
<tr>
<td>(0.002)</td>
<td>(0.002)</td>
<td>(0.002)</td>
<td>(0.002)</td>
<td>(0.002)</td>
<td>(0.002)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
</tr>
</tbody>
</table>

Notes: *The figures are the estimated slope coefficients obtained when the log of the quality-adjusted price-rent ratios are regressed on an intercept and the log of price or the log of rent. The figures in the brackets are the estimated standard errors.

Table 9: Quality-Adjusted Price-Rent Ratios for Different Market Segments

<table>
<thead>
<tr>
<th>Year</th>
<th>Price-Rent from Price Data</th>
<th>Price-Rent From Rent Data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Lower Quartile</td>
<td>Median Quartile</td>
</tr>
<tr>
<td>2003</td>
<td>27.39</td>
<td>29.17</td>
</tr>
<tr>
<td>2005</td>
<td>26.52</td>
<td>27.07</td>
</tr>
<tr>
<td>2006</td>
<td>23.88</td>
<td>26.02</td>
</tr>
<tr>
<td>2007</td>
<td>21.57</td>
<td>25.65</td>
</tr>
<tr>
<td>Average</td>
<td>23.10</td>
<td>25.24</td>
</tr>
</tbody>
</table>
Table 10: Robustness Checks

<table>
<thead>
<tr>
<th>Year</th>
<th>Base</th>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
<th>(d)</th>
<th>(e)</th>
<th>(f)</th>
<th>(g)</th>
<th>(h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2001</td>
<td>20.48</td>
<td>23.35</td>
<td>21.10</td>
<td>20.76</td>
<td>21.05</td>
<td>21.35</td>
<td>20.69</td>
<td>20.08</td>
<td>20.66</td>
</tr>
<tr>
<td>2004</td>
<td>29.78</td>
<td>32.11</td>
<td>30.38</td>
<td>29.90</td>
<td>29.10</td>
<td>29.96</td>
<td>30.05</td>
<td>29.53</td>
<td>29.96</td>
</tr>
<tr>
<td>2005</td>
<td>27.09</td>
<td>29.48</td>
<td>28.08</td>
<td>26.98</td>
<td>26.94</td>
<td>27.28</td>
<td>27.43</td>
<td>26.37</td>
<td>27.11</td>
</tr>
<tr>
<td>2006</td>
<td>25.44</td>
<td>27.76</td>
<td>26.21</td>
<td>25.36</td>
<td>25.89</td>
<td>25.63</td>
<td>25.51</td>
<td>24.84</td>
<td>25.44</td>
</tr>
<tr>
<td>2007</td>
<td>23.60</td>
<td>25.31</td>
<td>24.56</td>
<td>23.61</td>
<td>23.65</td>
<td>23.63</td>
<td>23.41</td>
<td>23.35</td>
<td>23.57</td>
</tr>
<tr>
<td>Average</td>
<td>24.70</td>
<td>27.00</td>
<td>25.55</td>
<td>24.81</td>
<td>24.76</td>
<td>24.90</td>
<td>24.83</td>
<td>24.36</td>
<td>24.75</td>
</tr>
</tbody>
</table>

Notes: (a) Obtained from running HM1 models on observations which have all three physical characteristics, without correcting for omitted variables bias. (b) Single imputation price-rent ratios—$P/R_{sold}^{SI}$ and $P/R_{rented}^{SI}$ (see section 1.2)—are estimated, instead of double imputation price-rent ratios. (c) Hedonic models in equations 6 and 7 are specified on price and rent levels instead of log of prices and rents. (d) $Med[P/R_{pooled}^{DI}]$ specified in section 1.3 is used. (e) Double imputation hedonic price and rent indexes are used in order to estimate $P_{st}$ and $R_{st}$ in equations 16 and 15, respectively (the main results use repeat-sales and -rents indexes). (f) For dwellings with multiple prices and rents, we consider each price-rent pair in order to obtain $P/R_{t}^{AM}$ (in equation 17) where the main results use only the chronologically closest price and rent observations. (g) The regressions required to obtain $\lambda_{t,HM8}$ (equation 18) are estimated using the sample HM8s\AMs (instead of using HM8s which includes AMs) and then the price-rent ratios of the AMS dwellings are imputed from the regression results. (h) The correction factor ($\lambda_{t,HMj|HM8}$ equation 19) is obtained using the HM1s, i.e. $\lambda_{t,HMj|HM8} = HMjm(HM1s)/HM8m(HM1s)$. 

Figure 1: Quality-Adjusted and Unadjusted Price-Rent Ratios

Figure 2: Price and Rent Indexes of Lower and Upper Ends of the Market
Figure 3: $\sigma$-Convergence