Chain-linking and seasonal adjustment of the quarterly national accounts

The method of chain-linking the quarterly national accounts was changed with the revised compilation of data for third quarter 2007, which was published 16 January 2008.

Method of chain-linking

In earlier compilations the quarterly chain-linked data was calculated with fourth quarter of the previous year as point of reference. This method is called quarterly overlap. The method used now calculates the chain-linked data with the annual data of the previous year as point of reference. This method is called annual overlap.

With this method it is no longer necessary to benchmark the quarterly chain-linked data to the annual chain-linked data, because the sum of the quarters equals the annual chain-linked value, when the annual overlap method is used.

In this paper it is shown how formulas for aggregation of and contribution to growth for quarterly chain-linked series can be derived from the formula for annual overlap chain-linking.

Indirect seasonal adjustment

When the annual overlap method is used, it is possible to use indirect seasonal adjustment for the chain-linked series. The seasonally adjusted chain-linked aggregates can be calculated as an aggregate of the seasonally adjusted components using the above mentioned formula for aggregation of annual overlap chain-linked series. For data in current prices indirect seasonal adjustment is also used in the Danish quarterly national accounts.

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Aggregation and contribution to growth

Annual overlap chain-linking

Chain-linked data in the Danish quarterly national accounts are calculated using the annual overlap method. The formula for an annual overlap chain-linked value $K$ in the quarter $q$ in year $t$ is

1) \[ K_t^q = \overline{K}_{t-1} \cdot \frac{D_t^q}{L_{t-1}}, \]

where $D$ is the value in previous years prices, $L$ is current prices and

\[ \overline{X}_t = \sum_{q=1}^{4} X_t^q \]

Aggregation of chain-linked series

1) can be rewritten to

2) \[ K_t^q \cdot \frac{L_{t-1}}{\overline{K}_{t-1}} = D_t^q \]

Because the normal rules of additivity applies for data in previous years prices, the aggregate $D_t^q$ equals the sum of its components

3) \[ D_t^q = \sum_i D_t^{iq}, \text{ where } i \text{ is an index of the subcomponents.} \]

When 2) is combined with 3)

\[ K_t^q \cdot \frac{L_{t-1}}{\overline{K}_{t-1}} = \sum_i \left( K_t^{iq} \cdot \frac{L_{t-1}^i}{\overline{K}_{t-1}^i} \right) \iff \]

\[ \sum_i \left( K_t^{iq} \cdot \frac{L_{t-1}^i}{\overline{K}_{t-1}^i} \right) = K_t^q \cdot \frac{L_{t-1}}{\overline{K}_{t-1}} \iff \]

4) \[ K_t^q = \frac{\sum_i \overline{P}_{t-1}^i \cdot K_t^{iq}}{\overline{P}_{t-1}^i}, \text{ where } \overline{P}_t^i = \frac{L_t^i}{\overline{K}_t^i} \]
Formula 4) can then be used to aggregate chain-linked quarterly series with use of the price indices from the previous year. This method of aggregation is used to create indirectly seasonally adjusted chain-linked series by aggregation of the seasonally adjusted subcomponents.

**Quarterly contributions to growth**

An expression for the contribution to growth ($\text{CG}_i^{qt}$) from a subcomponent $i$ can be derived from the above formula 4). The result will depend on the quarter, because the growth from fourth to first quarter depends on price deflators for two different years. An expression for the contribution to growth for the second, third and fourth quarter will therefore be derived at first, and afterwards it will be derived for the first quarter.

To calculate the quarterly contribution to growth for the second, third and fourth quarter ($q=2,3,4$) formula 4) is used as starting point

$$K_i^q - K_i^{q-1} = \frac{\sum_{i} P_{t-1} K_i^{iq}}{P_t} - \frac{\sum_{i} P_{t-1} K_i^{iq-1}}{P_t}$$

The above equation shows, that the growth in an aggregate (left side) can be seen as the sum of contributions from each of the subcomponents. The quarterly contribution to growth from the subcomponent $i$ ($\text{CG}_i^{qt}$) can therefore be calculated as

$$\text{CG}_i^{qt} = \frac{P_{t-1} K_i^{iq} - K_i^{iq-1}}{P_{t-1} K_i^{q-1}}$$

for $q = 2,3,4$.
The quarterly contribution to growth for the first quarter can in a similar manner be derived using formula 4)

\[ K_t^q = \frac{\sum P_{t-1}^i K_{t-1}^q}{P_{t-1}} \quad \land \quad K_{t-1}^{q-1} = \frac{\sum P_{t-2}^i K_{t-1}^{q-1}}{P_{t-2}} \Rightarrow \]

\[ K_t^q - K_{t-1}^{q-1} = \frac{\sum \left( P_{t-2}^i P_{t-1}^i K_t^q - P_{t-1}^i P_{t-2}^i K_{t-1}^{q-1} \right)}{P_{t-1} P_{t-2}} \equiv \]

\[ K_t^q - K_{t-1}^{q-1} = \frac{\sum \left( P_{t-2}^i P_{t-1}^i K_t^q - P_{t-1}^i P_{t-2}^i K_{t-1}^{q-1} \right)}{P_{t-1} P_{t-2} K_{t-1}^{q-1}} \]

Again an equation is derived where the growth in an aggregate (left side) equals a sum of contributions from each of the subcomponents. The quarterly contribution to growth in the first quarter from the subcomponent \( i \) (\( CG_{t,i}^q \)) can therefore be calculated as

\[ CG_{t,i}^q = \frac{P_{t-2}^i P_{t-1}^i K_t^q - P_{t-1}^i P_{t-2}^i K_{t-1}^{q-1}}{P_{t-1} P_{t-2} K_{t-1}^{q-1}} \equiv \]

\[ 6) \quad CG_{t,i}^q = \left( \frac{P_{t-1}^i}{P_{t-2}^i} \right) K_t^q \left( \frac{P_{t-2}^i}{P_{t-1}^i} \right) K_{t-1}^{q-1}, \text{ for } q = 1 \]

Both of the formulas for contribution to growth 5) and 6) looks a lot like normal contribution to growth calculations. The difference is that the contributions is weighed with the relationship between the price indices for the subcomponent and the aggregate in the previous year. The difference between 5) and 6) is, that the annual price index for the previous year changes from \( q-1 \) to \( q \), when the growth from fourth to first quarter is considered in 6).

The above method to calculate contributions to growth was derived using the formula for aggregation 4), where the chain-linked series is aggregated using price indices from the previous year. This was derived using the additivity of data in previous years prices, but an expression for a chain-linked aggregate as a function of its subcomponents can also be derived with the use of additivity in current prices

\[ 7) \quad L_t^q = \sum_i L_t^{iq} \]
Because \( L_i^q = P_i^q K_i^q, \) 7) can be rewritten to

\[
L_i^q = \sum_j L_j^q \quad \Leftrightarrow \quad P_i^q K_i^q = \sum_j P_i^q K_i^q \quad \Leftrightarrow \quad \sum_j P_i^q K_i^q = \frac{1}{P_i^q}
\]

8) \( K_i^q = \frac{1}{P_i^q} \)

Using 8) expressions for the contribution to growth for chain-linked quarterly series can be derived with use of the same methods that were used to derive 5) og 6). These expressions are shown below in 9) and 10)

\[
9) \quad CG_i^{q-1} = \left( \frac{P_i^{q-1}}{P_i^q} \right) K_i^q - \left( \frac{P_i^{q-1}}{P_i^q} \right) K_i^{q-1} \quad \text{for} \quad q = 2, 3, 4
\]

\[
10) \quad CG_i^{t-1} = \left( \frac{P_i^{t-1}}{P_i^q} \right) K_i^q - \left( \frac{P_i^{t-1}}{P_i^q} \right) K_i^{q-1} \quad \text{for} \quad q = 1
\]

In these alternative calculations of contribution to growth the price indices for the specific quarter is used as weights. Therefore the two terms in the numerator is always weighted using different price indices, which was only the case for the contribution for the first quarter growth with the formulas derived using previous years prices.

The two procedures using respectively the additivity of data in previous years prices and additivity in current prices can also be used in combination. For example for \( q = 1 \)

\[
K_i^q = \frac{1}{P_i-1} \quad \text{and} \quad K_i^{q-1} = \frac{1}{P_i-1} \frac{P_i^{q-1}}{P_i-1}
\]

Using the above expressions for \( K \) an expression for the contribution to growth for the first quarter could be derived, where the chain-linked values in the first quarter would be weighed with the annual price index for \( t-1 \), while the values in the fourth quarter of \( t-1 \) would be weighed with the price index of that fourth quarter. Only when using additivity in previous years prices for both quarters an expression for the contribution to growth with the same price index in both quarters can be obtained, as in formula 5). This is why formula 5) and 6) has been chosen to calculate the contributions to growth for the quarterly, seasonally adjusted growth in the danish quarterly national accounts.
Annual contributions to growth

Expressions for contributions to quarterly growth in an aggregate was calculated above. In this section contributions to annual growth in a quarterly aggregate will be calculated. That is the growth from quarter \( q \) in year \( t-1 \) to quarter \( q \) in year \( t \). This corresponds to the above derivations of quarterly contributions to growth for the first quarter, except that the same \( q \) is used in year \( t \) and \( t-1 \). Using 6) the expression 11) below is derived for the annual contribution to growth for a quarter \( q \) in year \( t \)

\[
11) \quad CG_{t}^{iq} = \frac{\left( \frac{P_{t-1}^i}{P_{t-1}^q} \right) K_{t-1}^{iq} - \left( \frac{P_{t-2}^i}{P_{t-2}^q} \right) K_{t-1}^{iq}}{K_{t-1}^{iq}}
\]

Alternatively 10) can be used instead

\[
12) \quad CG_{t}^{iq} = \frac{\left( \frac{P_{t}^{iq}}{P_{t}^{q}} \right) K_{t}^{iq} - \left( \frac{P_{t-1}^{iq}}{P_{t-1}^{q}} \right) K_{t-1}^{iq}}{K_{t-1}^{iq}}
\]

In both 11) and 12) price indices for two different years are used in the calculation, which results in, that these formulas for annual contributions to growth are inconsistent with the formula for calculating contributions to growth for annual chain-linked data as seen below in 13)

\[
13) \quad CG_{t}^{i} = \frac{P_{t}^{i}}{P_{t-1}} \frac{K_{t}^{i} - K_{t-1}^{i}}{K_{t-1}^{i}}
\]

But if a combination of the two approaches are used, as it was shown in the last section at page 5, a formula for annual contributions to growth for quarterly data can be calculated as in 14)

\[
14) \quad CG_{t}^{iq} = \frac{\left( \frac{P_{t-1}^i}{P_{t-1}^q} \right) K_{t}^{iq} - \left( \frac{P_{t-2}^i}{P_{t-2}^q} \right) K_{t-1}^{iq}}{K_{t-1}^{iq}}
\]

In 14) two different price indices are still used for the two quarters, but now they are from the same year. Another important property of 14) is, that because the first price indices are for the whole year and the others are for the specific quarter, formula 14) is consistent with 13). When calculating annual contributions to growth for the sum of all quarters in a year using 14) it results in the same as using 13). It is straightforward to show this algebraic by replacing \( K^q \) (and the corresponding price indices) with \( K \) in 14). Because of this consistency 14) has been chosen to calculate the annual contributions to growth for quarterly data in the Danish quarterly national accounts.